Extracting Hypernuclear Properties from the \((e, e'K^+)\) Cross Section

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ASTRA: Advanced and Open Problems in Low-Energy Nuclear and Hadronic STRAegeness Physics

ECT*, October 23-27, 2017
OUTLINE

* Motivation & disclaimer

* The \((e, e'K^+)\) cross-section
  ▶ Elementary cross-section
  ▶ Nuclear and hypernuclear dynamics
  ▶ Kinematics: combining \((e, e'p)\) and \((e, e'K^+)\)

* From nuclei to nuclear matter: the case of \(^{208}\text{Pb}\)

* Outlook
THE $A(e, e'K^+)_\Lambda A$ CROSS SECTION

★ Consider the process
\[
e(k) + A(p_A) \rightarrow e'(k') + K^+(p_K) + \Lambda A(p_{\Lambda A})
\]

★ Cross section ($i, j = 1, 2, 3$)
\[
d\sigma \propto L_{ij} W^{ij}
\]

▷ The lepton tensor $L_{ij}$ is fully specified by the measured electron kinematical variables

▷ The tensor $W^{ij}$, describing the nuclear response, contains all the information on both nuclear and hypernuclear dynamics
Lepton tensor

\[ L = \begin{pmatrix}
\eta_+ & 0 & -\sqrt{\epsilon_L \eta_+} \\
0 & \eta_- & 0 \\
-\sqrt{\epsilon_L \eta_+} & 0 & \epsilon_L
\end{pmatrix}, \]

\[ \eta_\pm = \frac{1}{2} (1 \pm \epsilon), \quad \epsilon = \left(1 + 2 \frac{|q|^2}{Q^2} \tan^2 \frac{\theta_e}{2}\right)^{-1}, \quad \epsilon_L = \frac{Q^2}{\omega^2} \epsilon \]

Target response tensor

\[ W^{ij} = \langle 0 | J_A^i (q) | F \rangle \langle F | J_A^j (q) | 0 \rangle \delta^{(4)} (q + p_0 - p_F) \]

Building blocks

\[ |0\rangle = |A\rangle, \quad J_A^i = \sum_{n=1}^{A} j^i(n), \quad |F\rangle = |K^+\rangle \otimes |\Lambda A\rangle \]

According to the paradigm of nuclear many-body theory, nuclear and hypernuclear states should be obtained from dynamical models based on phenomenological microscopic Hamiltonians.
Step 1: consider the elementary process

\[ e + p \rightarrow e' + \Lambda + K^+ \]

\[ K^+(p_K) \rightarrow \Lambda(p_\Lambda) \]

\[ \gamma(q) \rightarrow p(p_p) \]

\[ B(A) \]

\[ \Lambda(p_\Lambda) \]

\[ K^+(p_K) \rightarrow \Lambda(p_\Lambda) \]

\[ \gamma(q) \rightarrow p(p_p) \]

\[ S \]

\[ \gamma(q) \rightarrow p(p_p) \]

\[ M \]

\[ \gamma(q) \rightarrow p(p_p) \]

- *B, S and M* denote a nonstrange baryon, a strange baryon and a strange meson, respectively
Elementary cross section

\[ \frac{d\sigma_N}{dE_e' d\Omega_e' d\Omega_K} \propto \left[ \eta^+ W^{xx} + \eta^- W^{yy} + \epsilon_L W^{zz} + \sqrt{\epsilon_L \eta^+} (W^{xz} + W^{zx}) \right] \]

\[ W^{\mu\nu} \propto \sum_{spins} j^{\mu\dagger} j^\nu \]

example: s-channel

\[ j^\mu = \bar{u}(p_\Lambda) \Gamma(p_K) S_F(p_p + q) \Gamma^\mu(q) u(p_p) \]

\[ j^\mu = \sum_i A_i(s, t, u)\bar{u}(p_\Lambda)M_i^\mu u(p_p) \]

\[ s = (q + p_p)^2, \quad s = (q - p_K)^2, \quad s = (q - p_\Lambda)^2 \]

\[ M_1^\mu = \frac{1}{2} \gamma_5 [\not{q}, \gamma^\mu] \]

\[ M_2^\mu = \gamma_5 \left[ q^2 p_p^\mu - (q \cdot p_p)q^\mu \right], \quad M_3^\mu = \gamma_5 \left[ q^2 p_\Lambda^\mu - (q \cdot p_\Lambda)q^\mu \right] \]

\[ M_4^\mu = \gamma_5 \left[ \gamma^\mu (q \cdot p_p) - \not{q}p_p^\mu \right], \quad M_5^\mu = \gamma_5 \left[ \gamma^\mu (q \cdot p_\Lambda) - \not{q}p_\Lambda^\mu \right] \]

\[ M_6^\mu = \frac{1}{2} \gamma_5 [\not{q}q^\mu - \gamma^\mu q^2] \]

**The model parameters involved in the definition of the current, e.g. the strong coupling constants, are obtained from a fit to the existing data**
Step 2: the nuclear cross section: impulse approximation and factorization

At momentum transfer $|q|^{-1} \ll d$, $d \sim 1.5 \text{ fm}$ being the average nucleon-nucleon separation distance in the target nucleus, the beam particles interact with individual (bound, moving) nucleons

Within this scheme, the nuclear transition amplitude factorizes into the amplitude of the elementary process, a purely nuclear amplitude and a hypernuclear amplitude

The effects of Final State Interactions (FSI) between the outgoing $K^+$ and the recoiling system can be included using an optical potential
Nuclear Transition Amplitude

* Isolate the building blocks

\[ M_{0→F} = \langle K^+, Y A | J_A^i | 0 \rangle \]
\[ = \sum_n \sum_{k_p, k_Y} \left\{ \langle Y A | (A - 1)_n | Y \rangle \right\} \langle K^+ Y | j^i | p \rangle \left\{ \langle p | \langle (A - 1)_n | 0 \rangle \right\} \]

* Relation to the spectral function formalism of \((e, e'p)\)

\[ P_N(k_p, E_p) = \sum_n |\langle p | \langle (A - 1)_n | 0 \rangle|^2 \delta(E_p - E_n + E_0) \]
▷ probability of removing a proton of momentum \(k_p\) from the nuclear target, leaving the residual nucleus with energy \(E\)

\[ P_Y(k_Y, E_Y) = \sum_n |\langle Y | \langle (A - 1)_n | Y A \rangle|^2 \delta(E_Y - E_n + E_0) \]
▷ probability of removing the hyperon \(Y\), carrying momentum \(k_Y\) from the final state hypernucleus, leaving the residual nucleus with energy \(E\)
**Kinematics**

* Conservation of Energy \( \omega = E_e - E_{e'} \)

\[
\omega + M_A = E_{K^+} + E_{Y_A}
\]

▷ from the nuclear amplitude

\[
M_A = E_p + E_n
\]

▷ from the hypernuclear amplitude

\[
E_{Y_A} = E_Y + E_n
\]

* Missing energy \( E_{\text{miss}} = \omega - E_{K^+} \)

\[
\omega = E_{K^+} + E_Y - E_p \implies E_{\text{miss}} = E_Y - E_p
\]

* Note: in \((e, e'p)\)

\[
E_{\text{miss}}^{(e,e'p)} = -E_p \implies E_Y = E_{\text{miss}} + E_{\text{miss}}^{(e,e'p)}
\]
**Missing Energy Spectrum**

* Within the independent particle model

\[ P_N(k_p, E_p) \sim \sum_\alpha \delta(E_p - \epsilon_p^\alpha) \quad , \quad P_Y(k_Y, E_Y) \sim \sum_\alpha \delta(E_Y - \epsilon_Y^\alpha) \]

▷ \( P_\Lambda(k_\Lambda, E_\Lambda) \) (I. Vidaña, NPA 958(2017)48)

▷ \( P_N(k_p, E_p) \) from \((e, e'p)\)
**Missing Energy Spectrum of $^{9}\text{Be}(e, e'K^+)_\Lambda^{9}\text{Li}$**

★ M. Sotona and S. Frullani
PTP Supp. 117, 151 (1994)

★ JLab experiment E94-107
MODELING THE \((e, e'K^+)\) CROSS SECTION

* Bottom line: the nuclear cross section must be consistently obtained in the target rest frame

* The cross section of the elementary process \(e + p \rightarrow e' + \Lambda + K^+\) must properly account for both \textit{Fermi motion} and \textit{binding} of the struck proton, following the prescriptions successfully applied in analyses of \((e, e'p)\) data

* Requirements for a meaningful interpretation of the measured \(A(e, e'K^+)\Lambda\Lambda\) cross section
  ▶ A \textit{realistic} (i.e. beyond the mean-field approximation) description of the nuclear amplitude can be obtained combining \((e, e'p)\) data and the results of accurate nuclear matter calculation (Local Density Approximation)
  ▶ In principle, the full hypernuclear amplitude, determining the \(\Lambda\) spectral function, is also needed
  ▶ Assuming that hyperon states have negligible widths, the amplitude involving the final state hypernucleus can be estimated using the results of accurate calculations of the hyperon binding energy \(B_Y\).
FROM NUCLEI TO NUCLEAR MATTER AND NEUTRON STARS

Welcome to the Nuclear Charge Density archive
We have collected here data from Atomic and Nuclear Data Tables, Volumes 14, 36 and 60, which provide a variety of fits for nuclear charge density extracted from elastic electron-nucleus scattering. This webpage was created in order to have a digital collection of raw data online that could then be used to calculate the charge density using Sum of Gaussian, Fourier Bessel, or Charge Density distribution formulas.

Currently this webpage provides data files along with C++ code to calculate charge densities $\rho_{ch}$, and adjusted charge densities $(A/Z) * \rho_{ch}$.

- Alan Brody: Parameter Collection
- Bryan Lewis: C-code Development
- Stephen Washington: Web Page Creation

For questions, comments, or bug reports, please contact Donal Day by email: dbd [at] virginia [dot] edu
\[ ^{208}\text{Pb}(e, e'p)^{207}\text{Tl} \]

DATA

There are very similar spin-orbit partners. Therefore, only one of the two can be included in the fitting procedure. A transition energy has to be chosen, where the \( j = \frac{1}{2} \) strength vanishes and the \( j = \frac{3}{2} \) strength appears. For \( j = \frac{1}{2} \) to \( j = \frac{3}{2} \) states the transition energy was chosen such that ...

Figure 4.6 Results of the \( l \)-decomposition of the proton spectral function of \( ^{208}\text{Pb} \) in an excitation energy region from 0 to 25 MeV showing the spectroscopic strength in each bin as fraction \( n(\ell) \) of the sum-rule limit that is exhausted by the valence \( l \)-components (left) and by some deeper-lying \( l \)-components (right). The shown curves are calculated according to formula (4.4) and used to estimate the strength located outside the covered interval. The dashed lines separate excitation energy regions where different orbits were employed in the fit.
More $^{208}\text{Pb}(e, e'p)^{207}\text{Tl}$ Data

- Deviation of the observed binding energies from mean field predictions
- Widths of hole states
Spectroscopic Factors of $^{208}$Pb

NIKHEF results: $^{208}$Pb(e,e'p)$^{207}$Tl

$Z_{\alpha}/(2j+1)$

$Z^{NM}$

$n^{NM}$

$E_m$ [MeV]

Deep hole states largely unaffected by finite size and shell effects
Exploiting $K^+$ electroproduction data to constrain the models of hyperon dynamics requires a *quantitative* understanding of the nucleon sector. The $(e, e'p)$ cross section is the baseline for the extraction of hypernuclear properties from the $(e, e'K^+)$ cross section.

Information on hyperon interactions in neutron star matter can be obtained using heavy and neutron rich nuclei, whose deep-lying states are largely unaffected by surface and shell effects. The obvious choice is $^{208}\text{Pb}$, which has been extensively studied through the $(e, e'p)$ reaction.

In July, 2017, a letter of intent (LOI12-17-003: Studying $\Lambda$ interactions in nuclear matter with the $^{208}\text{Pb}(e, e'K^+)_{\Lambda}^{207}\text{Tl}$ reaction has been submitted to JLab PAC45.

From PAC45 report

**Summary:** The study of the $^{208}\text{Pb}(e,e'K^+)_{\Lambda}^{208}\text{Tl}$ reaction together with the other reactions proposed by this collaboration will eventually “complete” the systematic study of $\Lambda$ hypernuclear bound states over the wide mass range. The PAC recommends moving forward toward a full proposal.