The ANL-Osaka dynamical coupled-channels (DCC) approach for baryon spectroscopy

Hiroyuki Kamano
(KEK)

Collaborators:
T.-S. Harry Lee (Argonne Natl. Lab.)
Satoshi Nakamura (Cruzeiro do Sul U.)
Toru Sato (Osaka U.)

ECT* workshop on “Space-like and time-like electromagnetic baryonic transitions”
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\[
T^{(LSJ)}_{a,b}(p_a, p_b; E) = V^{(LSJ)}_{a,b}(p_a, p_b; E) + \sum_c \int_0^\infty q^2 dq V^{(LSJ)}_{a,c}(p_a, q; E)G_c(q; E)T^{(LSJ)}_{c,b}(q, p_b; E)
\]

- CC effect
- off-shell effect

\[
a, b, c = (\gamma^{(*)}N, \pi N, \eta N, \pi \Delta, \sigma N, \rho N, K \Lambda, K \Sigma, \cdots)
\]

\[
\pi \pi N
\]

✓ Summing up all possible transitions between reaction channels !!
  (⇒ satisfies multichannel two- and three-body unitarity)

e.g.) πN scattering

✓ Momentum integral takes into account off-shell rescattering effects in the intermediate processes.
Summing up all possible transitions between reaction channels satisfies multichannel two- and three-body unitarity.

Momentum integral takes into account off-shell rescattering effects in the intermediate processes.


\[
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\]

\[
a, b, c = \{N, N^*, \Delta, \ldots\}
\]

Summing over all possible reaction channels.

\[
\text{Stable channels: } MB = \pi N, \eta N, K\Lambda, K\Sigma
\]

\[
\text{Quasi 2-body channels: } MB = \pi \Delta, \rho N, \sigma N
\]

Exchange potentials

\[
\text{Z-diagrams}
\]

\[
\text{Bare } N^* \text{ states}
\]

\[
\text{Rescattering effects}
\]
ANL-Osaka DCC approach to N* & Δ*


\[
T^{(LSJ)}_{a,b}(p_a, p_b; E) = V^{(LSJ)}_{a,b}(p_a, p_b; E) + \sum_c \int_0^\infty q^2 dq V^{(LSJ)}_{a,c}(p_a, q; E)G_c(q; E)T^{(LSJ)}_{c,b}(q, p_b; E)
\]

\[a, b, c \quad = \quad (\gamma^{(*)}N, \pi N, \eta N, \{\pi \Delta, \sigma N, \rho N\}, K\Lambda, K\Sigma, \ldots)\]

\[\pi\pi N\]

✓ Summing up all possible transitions between reaction channels !!
(⇒ satisfies multichannel two- and three-body unitarity)

e.g.) πN scattering

✓ Momentum integral takes into account off-shell rescattering effects in the intermediate processes.
**ANL-Osaka DCC approach to N* & Δ**


**Why DCC approach ??**

- It defines **proper analytic structure** (branch points, cuts,...) of scattering amplitudes in the **complex energy plane**, as required by scattering theory

- Crucial for extracting resonances “correctly”, and avoiding **WRONG** resonance signals !!
  [e.g., Ceci et al, PRC84(2011)015205]

---

**Diagram:**

- **Im(W)** vs. **Re(W)**
- Points: πN, ππN, ηN, πΔ
- e.g. N*, Δ case
ANL-Osaka DCC approach to N* & Δ*


Why DCC approach ??

- It defines **proper analytic structure** (branch points, cuts,...) of scattering amplitudes in the **complex energy plane**, as required by scattering theory.

- Crucial for extracting resonances “correctly”, and avoiding **WRONG** resonance signals !! [e.g., Ceci et al, PRC84(2011)015205]

- Interested in clarifying the **physics of reaction dynamics** behind formation, substructure, etc. of baryon resonances.

Modeling reaction processes appropriately with a model Hamiltonian & solving proper quantum scattering equation (LS eq.) is crucial !!

**Diagram**

- Im(W) vs Re(W) for N*, Δ case
- Meson cloud representing quark-gluon “core”
- Dynamical origin of P11 N*

\[ T^{(LSJ)}_{a,b}(p_a, p_b; E) = V^{(LSJ)}_{a,b}(p_a, p_b; E) + \sum_{c} \int_{0}^{\infty} q^2 dq V^{(LSJ)}_{a,c}(p_a, q; E) G_c(q; E) T^{(LSJ)}_{c,b}(q, p_b; E) \]

Region our model can cover

\[ a, b, c = (\gamma^{(*)}N, \pi N, \eta N, \pi \Delta, \sigma N, \rho N, K \Lambda, K \Sigma, \cdots) \]

Latest published model:


Constructed by simultaneous analysis of

- \( \pi N \) SAID PW amps. \((W < 2.3 \text{ GeV})\)
- \( \pi \rho \rightarrow \eta N, K \Lambda, K \Sigma \) \((W < 2.1 \text{ GeV})\)
- \( \gamma \rho \rightarrow \pi N, \eta N, K \Lambda, K \Sigma \) \((W < 2.1 \text{ GeV})\)
- \( \gamma^{'}n^{'} \rightarrow \pi N \) \((W < 2 \text{ GeV})\)
ANL-Osaka DCC approach to $N^*$ & $\Delta^*$

$\gamma p \rightarrow \pi^0 p$

$\frac{d\sigma}{d\Omega}$ for $W < 2.1$ GeV

$\Sigma$ for $W < 2.1$ GeV

Red: Updated model [PRC94(2016)015201]
Blue: Original model [PRC88(2013)035209]

ANL-Osaka DCC approach to $N^*$ & $\Delta^*$

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Analysis of electroproduction reactions: Determining N-N*, N-Δ* e.m. transition form factors

N-N* e.m. transition form factors

γ* \rightarrow q \rightarrow N \rightarrow N*,Δ*
(q^2 = -Q^2)

How effective d.o.f. of baryon constituents changes with Q^2?

Q^2: small

Q^2: large

Meson electroproductions:

e \rightarrow γ* \rightarrow e' \rightarrow N \rightarrow N*, Δ*

CLAS database for 1π electroproductions
(Q^2 < 6 GeV^2)
Analysis of electroproduction reactions: Determining N-N*, N-Δ* e.m. transition form factors

\[
\frac{d\sigma}{dE_e d\Omega_e d\Omega_{\pi}} = \Gamma \left[ \sigma_T + \epsilon \sigma_L + \sqrt{2\epsilon(1+\epsilon)} \sigma_{LT} \cos \phi^*_\pi + \epsilon \sigma_{TT} \cos 2\phi^*_\pi + h_\epsilon \sqrt{2\epsilon(1-\epsilon)} \sigma_{LT}, \sin \phi^*_\pi \right].
\]

\[\sigma_T + \epsilon \sigma_L \text{ for } ep \rightarrow e\pi^0p\]

Data for structure functions are obtained with the help of K. Joo and L. C. Smith.

\[\sigma_\alpha = \sigma_\alpha(W, Q^2, \cos \theta^*_\pi)\]

Q² = 1.15 GeV², 1.10 < W < 1.69 GeV
Q² = 5.0 GeV², 1.11 < W < 1.69 GeV
Q² = 6.0 GeV², 1.11 < W < 1.39 GeV

\[\cos \theta\]
Extracted $\gamma(*) \, p \rightarrow N^*, \Delta^*$ transition form factors at finite $Q^2$ (evaluated at resonance poles)

Definition of $A_{1/2}$, $A_{3/2}$, $S_{1/2}$:

$$A_{3/2}(Q^2) = \langle R | j_{c.m.}^{\mu}(Q^2) \epsilon_\mu^{(\lambda_N = +1)} | N(\lambda_N = -1/2) \rangle$$

$$A_{1/2}(Q^2) = \langle R | j_{c.m.}^{\mu}(Q^2) \epsilon_\mu^{(\lambda_N = +1)} | N(\lambda_N = +1/2) \rangle$$

$$S_{1/2}(Q^2) = \langle R | j_{c.m.}^{\mu}(Q^2) \epsilon_\mu^{(\lambda_N = 0)} | N(\lambda_N = -1/2) \rangle$$
\( \gamma d \rightarrow \eta p n \) reaction:

Study of \( \eta N \) scattering parameters

At \( E_\gamma = 940 \) MeV & \( \theta_p = 0 \) deg.:

- \( \eta n \rightarrow \eta n \) rescattering

\[
R = \frac{\frac{d^3\sigma_{\text{full}}}{dM_{\eta n}d\Omega_p}|_{\theta_p=0^\circ}}{\frac{d^3\sigma_{\text{imp}}}{dM_{\eta n}d\Omega_p}|_{\theta_p=0^\circ}}
\]

Nakamura, HK, Ishikawa, arXiv:1704.07029
γd→ηpn reaction: Study of ηN scattering parameters

At \( E_\gamma = 940 \text{ MeV} \) & \( \theta_p = 0 \text{ deg.} \):

\[ \eta n \rightarrow \eta n \text{ rescattering} \]

\[ R = \frac{d^3\sigma_{\text{full}}/dM_{\eta n}d\Omega_p|_{\theta_p=0^\circ}}{d^3\sigma_{\text{imp}}/dM_{\eta n}d\Omega_p|_{\theta_p=0^\circ}} \]

With 5% accuracy of \( R \) (accessible with forthcoming experiment at ELPH@Tohoku U.), uncertainties in real parts of \( a_{\eta n}, r_{\eta n} \) can be drastically reduced:

\[ \text{Re}[a_{\eta n}] = 0.45\text{fm} +/- 0.5\text{fm} \Rightarrow ... +/- 0.1\text{fm} \]
\[ \text{Re}[r_{\eta n}] = -2.5\text{fm} +/- 3.5\text{fm} \Rightarrow ... +/- 0.5\text{fm} \]

Range from previous estimations

Nakamura, HK, Ishikawa, arXiv:1704.07029
γd → ηpn reaction:
Study of ηN scattering parameters

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At $E_\gamma = 940$ MeV & $\theta_p = 0$ deg.:

Nakamura, HK, Ishikawa, arXiv:1704.07029

Reliable model that all the relevant subprocesses are under control is very important to have this conclusion!!

$R = \frac{d^3\sigma_{\text{full}}/dM_{\eta n}d\Omega_p|_{\theta_p=0^\circ}}{d^3\sigma_{\text{imp}}/dM_{\eta n}d\Omega_p|_{\theta_p=0^\circ}}$

With 5% accuracy of $R$ (accessible with forthcoming experiment at ELPH@Tohoku U.), uncertainties in real parts of $a_{\eta n}$, $r_{\eta n}$ can be drastically reduced:

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Range from previous estimations
✓ Reliable neutrino reaction model is necessary for **precise** determination of neutrino parameters from **future** neutrino-oscillation experiments (leptonic CP phase, neutrino mass hierarchy...)

Neutrino-nucleon/nucleus reactions

\[ \nu, l, \nu, W^{+/0}, Z^0 \rightarrow V - A \rightarrow N, A, M(\text{meson}), B(\text{baryon}), \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma, \omega N, ... \]
**Application to neutrino-induced reactions**

- Reliable neutrino reaction model is necessary for precise determination of neutrino parameters from future neutrino-oscillation experiments (leptonic CP phase, neutrino mass hierarchy...)

Collaboration@J-PARC Branch of KEK Theory Center

Y. Hayato (ICRR, U. of Tokyo), M. Hirai (Nippon Inst. Tech.)
H. Kamano (KEK), S. Kumano (KEK)
S. Nakamura (Cruzeiro do Sul U.),
K. Saito (Tokyo U. of Sci.), M. Sakuda (Okayama U.),
T. Sato (Osaka U.)

[Rept. Prog. Phys. 80(2017)056301]
Application to neutrino-induced reactions

✓ Reliable neutrino reaction model is necessary for **precise** determination of neutrino parameters from future neutrino-oscillation experiments (leptonic CP phase, neutrino mass hierarchy…)

✓ Can provide **axial** transition form factors!!

Collaboration@J-PARC Branch of KEK Theory Center [http://j-parc-th.kek.jp/html/English/e-index.html]

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[⇒ Rept. Prog. Phys. 80(2017)056301]
Application to neutrino-induced reactions

The first-time full coupled-channels calculation of \(\nu\)-nucleon reactions beyond the \(\Delta(1232)\) region !!

- **Single pion production:**
  - \(\nu_{\mu}p \rightarrow \mu^- \pi^+ p\)
  - \(\nu_{\mu}n \rightarrow \mu^- \pi^0 p\)
  - \(\nu_{\mu}n \rightarrow \mu^- \pi^+ n\)

- **Double pion production:**

- **K\(\Lambda\) production:**
  - \(\nu_n \rightarrow \mu^- K^+ \Lambda\)

## NOTE: \(Q^2\) dependence of all N-N\(^*\) axial transition form factors are currently fixed with the nucleon dipole form factor.

## \# \# \(\eta\)N & K\(\Sigma\) productions can also be calculated.
Recently, our DCC approach has been applied to the analysis of $K^-p$ reactions to establish the mass spectrum of $Y^*$ (=$\Lambda^*$, $\Sigma^*$) baryons with strangeness -1. 


- Taking into account $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\eta\Lambda$, $K\Xi$, $\pi\Sigma^*(\pi\pi\Lambda)$, $\bar{K}^*N(\pi\bar{K}N)$ channels.

- Comprehensive analysis of *ALL* available data (more than 17,000 data points) of $K^-p \rightarrow \bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\eta\Lambda$, $K\Xi$ up to $W = 2.1$ GeV.

- Determination of threshold parameters (scattering lengths, effective ranges, ...); the partial-wave amplitudes of $\bar{K}N \rightarrow \bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\eta\Lambda$, $K\Xi$ for $S$, $P$, $D$, and $F$ waves.

- Extraction of $Y^*$ resonance parameters (mass, width, couplings, ...) defined by poles of scattering amplitudes.
Selected results of the fits

\[ K^- p \rightarrow K^- p \]
\[ \frac{d\sigma}{d\Omega} (1464 < W < 2100 \text{ MeV}) \]

\[ K^- p \rightarrow \pi^+ \Sigma^+ \]
\[ \frac{d\sigma}{d\Omega} (1536 < W < 2088 \text{ MeV}) \]

HK, Nakamura, Lee, Sato, PRC90(2014)065204

\[ K^- p \rightarrow \pi^0 \Lambda \]
\[ \frac{d\sigma}{d\Omega} (1536 < W < 2088 \text{ MeV}) \]

\[ P (1730 < W < 2080 \text{ MeV}) \]
\[ P (1689 < W < 1957 \text{ MeV}) \]

Red curve: Model A
Blue curve: Model B

\[ P \times \frac{d\sigma}{d\Omega} (1535 < W < 1696 \text{ MeV}) \]
\[ P \times \frac{d\sigma}{d\Omega} (1536 < W < 1696 \text{ MeV}) \]
Λ* and Σ* mass spectrum extracted from the K⁻p reaction data

Spectrum for Y* resonances found above the KN threshold

Red: Model A
Blue: Model B
Green: KSU [PRC88(2013)035205]
Black: PDG (only 4- & 3-star Y*; Breit-Wigner)

$M_R$: Resonance pole mass (complex)

$\text{Re}(M_R)$ (“width”)

$-2\text{Im}(M_R)$

$J^P(L_I 2J)$

“Λ” resonance (I=0)

“Σ” resonance (I=1)

Red: Model A
Blue: Model B
Green: KSU [PRC88(2013)035205]
Black: PDG (only 4- & 3-star Y*; Breit-Wigner)

Spectrum for Y* resonances found above the KN threshold

HK, Nakamura, Lee, Sato, PRC92(2015)025205
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\[-2\text{Im}(M_R)\] (“width”) 
\[\text{Re}(M_R)\]

\[M_R\]: Resonance pole mass (complex)

J^P(L_{12J}) “Λ” resonance (I=0)

J^P(L_{12J}) “Σ” resonance (I=1)

HK, Nakamura, Lee, Sato, PRC92(2015)025205
Λ* and Σ* mass spectrum extracted from the K⁻p reaction data

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New narrow 3/2+ resonance
M = 1671 – 5i MeV near the ηΛ threshold !!

HK, Nakamura, Lee, Sato, PRC92(2015)025205

\[ \text{Re}(M_R) - 2\text{Im}(M_R) \] ("width")

\[ M_R : \text{Resonance pole mass (complex)} \]
Λ* and Σ* mass spectrum extracted from the K⁻p reaction data

Spectrum for Y* resonances found above the KN threshold

Red: Model A
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J²P(LI²J²)

“Λ” resonance (l=0)

J²P(LI²J²)

“Σ” resonance (l=1)

Re(Mᵣ) -2Im(Mᵣ)

Mᵣ: Resonance pole mass (complex)

HK, Nakamura, Lee, Sato, PRC92(2015)025205

New narrow 3/2⁺ resonance
M = 1671 – 5i MeV
near the ηΛ threshold !!

Model A

Model B

P03 off

New narrow P03 resonance found in Model B is responsible for the angular dependence of dσ/dΩ !!
\( \Lambda^* \) and \( \Sigma^* \) mass spectrum extracted from the \( K^-p \) reaction data

Spectrum for \( Y^* \) resonances found above the \( \bar{K}N \) threshold

Red: Model A  
Blue: Model B  
Green: KSU[PRC88(2013)035205]  
Black: PDG (only 4- & 3-star \( Y^* \); Breit-Wigner)

New narrow \( 3/2^+ \) resonance  
\( M = 1671 - 5i \) MeV near the \( \eta \Lambda \) threshold!!

Spin partner of \( \Lambda(1520)3/2^- \) ??

HK, Nakamura, Lee, Sato, PRC92(2015)025205
A* and Σ* mass spectrum extracted from the K-p reaction data

Spectrum for Y* resonances found above the KN threshold

Red: Model A
Blue: Model B
Green: KSU[PRC88(2013)035205]
Black: PDG (only 4- & 3-star Y*; Breit-Wigner)

New narrow 3/2+ resonance
M = 1671 – 5i MeV near the ηΛ threshold!!

Spin partner of Λ(1520)3/2− ??

Low-lying Σ* resonances (PDG)
Σ(1193) 1/2+ ****
Σ(1385) 3/2+ ****
Σ(1480) *
Σ(1560) **
Σ(1580) 3/2− *
Σ(1620) 1/2− **
Σ(1660) 1/2+ ***
Σ(1670) 3/2− ****

HK, Nakamura, Lee, Sato, PRC92(2015)025205
$K^-d \to (\pi\Sigma)_0n$ reaction: Study of $\Lambda(1405)$ & other low-lying $Y^*$ resonances

$p_{K^-} = 1$ GeV, $\theta_n = 0$ deg.

Model B predicts two $J^P=1/2^-$ $\Lambda$ resonances below $\bar{K}N$ threshold:

$M_R = 1428 - i31$ MeV $\leftarrow$ corresponding to $\Lambda(1405)$

$M_R = 1397 - i98$ MeV

HK, Lee, PRC94(2016)015201
The reaction $K^- d \rightarrow (\pi \Sigma)^0 n$ study of $\Lambda(1405)$ and other low-lying $Y^*$ resonances.

$p_K^- = 1$ GeV, $\theta_n = 0$ deg.

Full (Model B for $\bar{K}N$ processes) w/o $J^P = 1/2^- \Lambda$ resonant amplitude in $\bar{K}exN \rightarrow \pi \Sigma$.

Model B predicts two $J^P = 1/2^- \Lambda$ resonances below $\bar{K}N$ threshold:

$M_R = 1428 - i31$ MeV $\leftrightarrow$ corresponding to $\Lambda(1405)$
$M_R = 1397 - i98$ MeV

Kawasaki-san's talk@MENU2016
**K^- d \rightarrow (\pi \Sigma)_0 n reaction: Study of \Lambda(1405) & other low-lying Y^* resonances**

\[ p_{K^-} = 1 \text{ GeV}, \quad \theta_n = 0 \text{ deg.} \]

HK, Lee, PRC94(2016)015201

Full (Model B for \( K^0 N \) processes)

\[ w/o \, J^P=1/2^- \Lambda \text{ resonant amplitude in } K^\text{ex} N \rightarrow \pi \Sigma \]

Model B predicts two \( J^P=1/2^- \Lambda \) resonances below \( K^0 N \) threshold:

\[ M_R = 1428 - i31 \text{ MeV} \leftarrow \text{corresponding to } \Lambda(1405) \]
\[ M_R = 1397 - i98 \text{ MeV} \]

**“K-exchange” term**

\[ = \text{(off-energy-shell) amp. from our DCC model} \]
Prospects for studies of baryon transition form factors

$N^* \text{ & } \Delta^*$ e.m. transition form factors ($Q^2 > 0$)

CLAS database for $1\pi$ electroproductions
($Q^2 < 6 \text{ GeV}^2, W < 2 \text{ GeV}$)
[ongoing]

+ $K^+\Lambda, K^+\Sigma^0$ electroproduction data
($Q^2=0.65-2.55 \text{ GeV}^2, W=1.65-2.25 \text{ GeV}$)
Prospects for studies of baryon transition form factors

N* & Δ* e.m. transition form factors ($Q^2 > 0$)

CLAS database for 1π electroproductions ($Q^2 < 6 \text{ GeV}^2$, $W < 2 \text{ GeV}$ )
[ongoing]

+ $K^+\Lambda, K^+\Sigma^0$ electroproduction data ($Q^2=0.65-2.55 \text{ GeV}^2$, $W=1.65-2.25 \text{ GeV}$)

Λ* & Σ* e.m. transition form factors ($Q^2 < 0$, $Q^2 = 0$)

\[
\begin{align*}
\text{K}^- & \quad \text{γ*} & \quad \text{γ} \\
\text{p} & \quad \text{Λ}, \Sigma & \quad \text{Λ*}, \Sigma^* \\
\text{Λ*}, \Sigma^* & \quad \ell^+ & \quad \ell^- \\
\text{p} & \quad \text{Λ}, \Sigma & \quad \text{Λ*}, \Sigma^* \\
\text{K}^- & \quad \text{γ} & \quad \text{γ*}
\end{align*}
\]
Prospects for studies of baryon transition form factors

**N* & Δ* e.m. transition form factors (Q^2 > 0)**

CLAS database for 1π electroproductions (Q^2 < 6 GeV^2, W < 2 GeV)
[ongoing]

+ K+Λ, K+Σ^0 electroproduction data (Q^2=0.65-2.55 GeV^2, W=1.65-2.25 GeV)

**Λ* & Σ* e.m. transition form factors (Q^2 < 0, Q^2 = 0)**

**WEAK** current transition form factors (Q^2 < 0)

- Axial transition form factors

- |ΔS|=1(u→s), |ΔC|=1(d→c) transition form factors
Thank you for your attention !!
Back up
ANL-Osaka DCC approach to N* & Δ*


**γp → π^0p**

\[
\frac{d\sigma}{d\Omega} \text{ for } W < 2.1 \text{ GeV}
\]

**γp → K+Λ**

\[
\frac{d\sigma}{d\Omega} \text{ for } W < 2.1 \text{ GeV}
\]

**πp → K^0Σ^0**

\[
\frac{d\sigma}{d\Omega} \text{ for } W < 2.1 \text{ GeV}
\]

Red: PRC94(2016)015201 (updated ver.)

Blue: PRC88(2013)035209
Comparison of $N^*$ & $\Delta^*$ spectrum between multichannel analyses

HK, Nakamura, Lee, Sato, PRC88 (2013) 035209

Spectrum for low-lying states with $\text{Re}(M_R) < 1.6$ GeV is now well established !!
(One exception is $2^{\text{nd}}$ P33, Roper-like state of $\Delta$)

$\text{Re}(M_R)$
-2$\text{Im}(M_R)$
($\text{"width"}$)

$M_R$: Resonance pole mass (complex)

PDG: 4* & 3* states assigned by PDG2012
AO: ANL-Osaka (DCC)
J: Juelich [DCC, EPJA49(2013)44]
Extracted scattering lengths and effective ranges

HK, Nakamura, Lee, Sato, PRC90(2014)065204

### Scattering length and effective range

<table>
<thead>
<tr>
<th></th>
<th>Model A</th>
<th>Model B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( I = 0 )</td>
<td>( I = 1 )</td>
</tr>
<tr>
<td>( a_{Kp} ) (fm)</td>
<td>(-1.37 + i0.67)</td>
<td>(0.07 + i0.81)</td>
</tr>
<tr>
<td>( a_{\eta\Lambda} ) (fm)</td>
<td>(1.35 + i0.36)</td>
<td>(-)</td>
</tr>
<tr>
<td>( a_{K\Xi} ) (fm)</td>
<td>(-0.81 + i0.14)</td>
<td>(-0.68 + i0.09)</td>
</tr>
<tr>
<td>( r_{Kp} ) (fm)</td>
<td>(0.67 - i0.25)</td>
<td>(1.01 - i0.20)</td>
</tr>
<tr>
<td>( r_{\eta\Lambda} ) (fm)</td>
<td>(-5.67 - i2.24)</td>
<td>(-)</td>
</tr>
<tr>
<td>( r_{K\Xi} ) (fm)</td>
<td>(-0.01 - i0.33)</td>
<td>(-0.42 - i0.49)</td>
</tr>
</tbody>
</table>

\[ a_{K-p} = -0.65 + i0.74 \text{ fm (Model A)} \]
\[ a_{K-p} = -0.65 + i0.76 \text{ fm (Model B)} \]
N → "1st P33(J^P=3/2^+) Δ" transition form factor A_{3/2}
[evaluated at Δ pole mass: M_R = 1210 –i 50 MeV]

- Current = πN, ππN, ηN, KΛ, KΣ; 2 bare states in P33
- JLMS = πN, ππN, ηN; 2 bare states in P33
- Sato-Lee = πN; 1 bare state in P33