The NLO dilepton rate meets the lattice

Jacopo Ghiglieri, AEC ITP, University of Bern

ECT* workshop 2015, Trento, December 3 2015
In this talk:

- the *thermal photon* and *dilepton* rates at NLO in an infinite, equilibrated medium in different kinematical regimes
  - at zero virtuality JG Hong Kurkela Lu Moore Teaney *JHEP1305* (2013)
  - at small virtuality JG Moore *JHEP1412* (2014)
  - at larger virtuality Ghisoiu Laine *JHEP1410* (2014), JG Moore
- a comparison with lattice data JG Kaczmarek Laine Meyer
Basics of e/m production

- $\alpha \ll 1$ implies that real/virtual photon production are rare events and that rescatterings and back-reactions are negligible: medium is transparent to/not cooled by EM radiation

- At leading order in QED and to all orders in QCD the photon and dilepton rates are given by (in eq.)

$$\frac{d\Gamma_{\gamma}(k)}{d^3 k} = -\frac{\alpha}{4\pi^2 k} n_B(k^0) \rho_{EM}(k)$$

$$\frac{d\Gamma_{l^+l^-}(K)}{dk^0 d^3 k} = -\frac{\alpha^2}{6\pi^3 K^2} n_B(k^0) \rho_{EM}(K)$$

thermal distribution x spectral function of the EM current
pQCD: QCD action (and EFTs thereof), thermal average can be generalized to non-equilibrium. Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$
**Theory approaches**

- **pQCD**: QCD action (and EFTs thereof), thermal average can be generalized to non-equilibrium. **Real world**: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$

- **lattice QCD**: Euclidean QCD action, pure thermal average. **Real world**: analytically continue to Minkowskian domain
Theory approaches

- **pQCD:** QCD action (and EFTs thereof), thermal average can be generalized to non-equilibrium. **Real world:** extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$

- **lattice QCD:** Euclidean QCD action, pure thermal average. **Real world:** analytically continue to Minkowskian domain

- **AdS/CFT:** $\mathcal{N}=4$ action, in and out of equilibrium, weak and strong coupling. **Real world:** extrapolate to QCD
Motivation

- Test the reliability of the perturbative rates
Motivation

• Test the reliability of the perturbative rates
• by going to NLO
Motivation

- Test the reliability of the perturbative rates
  - by going to NLO
  - by interplay with lattice measurements
Motivation

• Test the reliability of the perturbative rates
  • by going to NLO
  • by interplay with lattice measurements
• Phenomenological motivation clear
Motivation

• Test the reliability of the perturbative rates
  • by going to NLO
  • by interplay with lattice measurements

• Phenomenological motivation clear

• More theoretical motivation: lots of knowledge about perturbative thermodynamics to high orders, not so much about dynamical quantities. Is convergence better/worse?
Kinematics of e/m production

\[
\frac{d\Gamma_{l^+l^-}(K)}{dk^0 d^3 k} = -\frac{\alpha^2}{6\pi^3 K^2} n_B(k^0) \rho_{EM}(K)
\]
Consider $k^0 + k \sim T$  $k^0 - k \sim g^2 T$
NLO at small $K^2$

- Consider $k^0 + k \sim T$, $k^0 - k \sim g^2 T$. Includes real photons

- A phenomenological motivation: low-mass dileptons as an ersatz real photon measurement (see for instance PHENIX). Is the spectral function smooth approaching the light cone?
At zeroth order \((\alpha_{\text{EM}} g^0)\): 

Apparently LO, but very small phase space, proportional to \(K^2 \sim g^2 T^2\). This is a collinear process.

Leading order is \(\alpha_{\text{EM}} g^2\). Processes where a quark is kicked off the mass shell also LO

Strength of the kick (virtuality) determines the momentum region of the calculation

\[
\frac{d\Gamma_{l^+l^-}(K)}{dk^0d^3k} = -\frac{\alpha^2}{6\pi^3 K^2} n_B(k^0) \rho_{\text{EM}}(K)
\]

\[
J^\mu = \sum_{q=uds} e_q \bar{q} \gamma^\mu q
\]
Kinematical regions

- Define a light-cone ($k$ along $z$)
  
  $$P = (p^+, p^-, p_\perp) \quad p^+ = (p^0 + p^z)/2 \quad p^- = p^0 - p^z$$

- Momentum conservation at the current insertion gives three regions
  
  $$J^\mu = \sum_{q=uds} e_q \bar{q} \gamma^\mu q : \sim \blacktriangleleft$$

- Hard off-shell

- Soft, smaller phase space but enhancement

- Collinear, both nearly on shell and enhanced
• In the \((p^+, p_\perp)\) plane \((P = \text{quark loop momentum})\)

\[
P = (p^+, p^-, p_\perp)
\]

Kinematical regions

- **Hard** \(P = (T, T, T), P^2 \sim T^2\)
- **Soft** \(P = (gT, gT, gT), P^2 \sim g^2T^2\)
- **Collinear** \(P = (T, gT, T), P^2 \sim gT^2\)

\[
p^+ = \frac{(p^0 + p^z)}{2} \quad p^- = p^0 - p^z
\]
The $2\leftrightarrow 2$ region

- Two loop diagrams ($\alpha_{EM}g^2$)

where the cuts correspond to the so-called $2\leftrightarrow 2$ processes (with their crossings and interferences):

- IR divergence (Compton) when $t$ goes to zero
• The IR divergence is cured by a proper resummation in the soft sector through the Hard Thermal Loop effective theory. Braaten Pisarski NPB337 (1990)
Introducing the soft scale

- The IR divergence is cured by a proper resummation in the soft sector through the **Hard Thermal Loop** effective theory [Braaten Pisarski NPB337 (1990)]

- The Landau cut of the HTL propagator opens up the phase space in this (apparently one-loop) diagram

\[
\begin{align*}
K & \propto \ln(\Lambda_{\text{UV}} m_{\infty}) + \# \\
\text{Soft: HTL} & + \ln(k_0 T) \\
\text{Hard: Bare} & = \ln(k_0 T m_{\infty}) + \# \end{align*}
\]

\[\text{Bare} \leftarrow \text{HTL} \rightarrow \text{Bare}\]
Introducing the soft scale

- The IR divergence is cured by a proper resummation in the soft sector through the **Hard Thermal Loop** effective theory. Braaten Pisarski *NPB337* (1990)

- The Landau cut of the HTL propagator opens up the phase space in this (apparently one-loop) diagram.

- In the end one obtains the result

\[
\left. \frac{d\Gamma}{d^3k} \right|_{2\leftrightarrow2} \propto e^2 g^2 \left[ \log \frac{T}{m_\infty} + C_{2\leftrightarrow2} \left( \frac{k}{T} \right) \right]
\]

The dependence on the cutoff cancels out.

Collinear processes

- These diagrams contribute to LO if small $(g)$ angle radiation/annihilation [Aurenche Gelis Kobes Petitgirard Zaraket 1998-2000]
- Virtual photon formation times is then of the same order of the soft scattering rate $\Rightarrow$ interference: *LPM effect*
- Requires resummation of infinite number of ladder diagrams

\[
\frac{d\Gamma_{l+\ell^-}}{dk^0 d^3k}\bigg|_{\text{coll}} = \text{Re} \left( \begin{array}{c}
\end{array} \right)
\]

Quark statistical fncts $\times$ DGLAP $\times$ transverse evolution
\[
\frac{d\Gamma}{d^4 K} = \frac{4\alpha}{3\pi^3 K^2} \int \frac{dp^+}{2\pi} n_F(k^+ + p^+)[1 - n_f(p^+)] \left[ \frac{(p^+)^2 + (p^+ + k^+)^2}{2(p^+ + k^+)^2} \right] \lim_{x_\perp \to 0} \text{Re} \nabla_{x_\perp} f(x_\perp) \bigg] + \frac{2k^-}{k^+} \lim_{x_\perp \to 0} \text{Re} g(x_\perp)
\]

$x^+ \gg x_\perp \gg x^-$

$1/g^2 T \gg 1/g T \gg 1/T$
LPM resummation

- Quark statistical fncts × DGLAP × transverse evolution

$$\frac{d\Gamma}{d^4K} = \frac{4\alpha}{3\pi^3 K^2} \int \frac{dp^+}{2\pi} n_F(k^+ + p^+)[1 - n_f(p^+)] \left[ \frac{(p^+)^2 + (p^+ + k^+)^2}{2(p^+(p^+ + k^+))^2} \lim_{x_\perp \to 0} \text{Re} \nabla_{x_\perp} f(x_\perp) + \frac{2k^-}{k^+} \lim_{x_\perp \to 0} \text{Re} g(x_\perp) \right]$$

- Transverse diffusion and Wilson-loop correlators evolve the transverse density $f$ along the spacetime light-cone

$$-2i\nabla \delta^2(x_\perp) = \left[ \frac{ik^+}{2p^+(p^++k^+)} \left( m_\infty^2 - \nabla_{x_\perp}^2 \right) + k^- + C(x_\perp) \right] f(x_\perp)$$

$x^+ \gg x_\perp \gg x^-$

$1/g^2T \gg 1/gT \gg 1/T$

**LPM resummation: two inputs**

- Asymptotic mass \( m_\infty^2 = 2g^2 C_R \left( \int \frac{d^3 p}{(2\pi)^3} \frac{n_B(p)}{p} + \int \frac{d^3 p}{(2\pi)^3} \frac{n_F(p)}{p} \right) \)

- Light-cone Wilson loop, related to \( \hat{q} \)

\[
\hat{q} \equiv \int_0^{q_{\text{max}}} \frac{d^2 q_\perp}{(2\pi)^2} q_\perp^2 C(q_\perp)
\]

\[
\propto e^{C(x_\perp)} L
\]

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D’Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- Soft contribution becomes Euclidean! Caron-Huot \textbf{PRD79} (2008), can be “easily” computed in perturbation theory

Sources of NLO corrections

- As usual in thermal field theory, the soft scale $gT$ introduces NLO $O(g)$ corrections.
- The soft region and the collinear region both receive $O(g)$ corrections.
- There is a new semi-collinear region.
- The NLO calculation is still not sensitive to the magnetic scale $g^2T$. 
NLO regions

\[ P = (T, gT, \sqrt{gT}), \quad P^2 \sim gT^2 \]
Euclideanization of light-cone soft physics

For $v = x_z/t = \infty$ correlators (such as propagators) are the equal time Euclidean correlators.

$$G^>(t = 0, x) = \sum_p G_E(\omega_n, p) e^{ip \cdot x}$$

- Causality: retarded functions analytic for positive imaginary parts of all timelike and lightlike variables: the above result can be extended to the lightcone

$$G^>(t = x_z, x_\perp) = \sum_p G_E(\omega_n, p_\perp, p_z + i\omega_n) e^{i(p_\perp \cdot x_\perp + p_z x_z)}$$

- The sums are dominated by the zero mode for soft physics $\Rightarrow$ EQCD!

- Equivalent to sum rules Caron-Huot PRD79 (2009)
The collinear sector

- Four sources of $O(g)$ corrections

- $m^2_\infty$ at NLO, Caron-Huot PRD79 (2009) 125002

\[
\delta m^2_\infty = 2g^2C_RT \int \frac{d^3q}{(2\pi)^3} \left( \frac{1}{q^2 + m^2_D} - \frac{1}{q^2} \right) = -g^2C_RTm_D \frac{Tm_D}{2\pi}
\]

- $c(x_\perp)$ at NLO $\Rightarrow$ one-loop rungs Caron-Huot PRD79 (2009) 065039

$p^+ \sim gT$ or $p^+ + k \sim gT$. Mistreated soft limit

$p_\perp \sim \sqrt{gT}, p^- \sim gT$. Mistreated semi-collinear limit

Identify and subtract the limiting behaviors thereof
The NLO soft region

- 4 diagrams with HTL vertices and propagators on the soft line
- Could brute-force them numerically. Or think again about analyticity, light-cones
The soft region: sum rules

- We have found the fermionic analogue of the Aurenche Gelis Zaraket JHEP0205 (2002) sum rule

- The leading-order soft contribution ($P_{\text{soft}}$)

\[
(2\pi)^3 \frac{d\Gamma}{d^3 k_{\text{soft}}} \propto \int \frac{dp^+ d^2 p_\perp}{(2\pi)^3} \text{Tr} \left[ \gamma^- (S_R(P) - S_A(P)) \right] \bigg|_{p^- = 0}
\]

where

\[
S(P) = \frac{1}{2} \left[ (\gamma^0 - \vec{\gamma} \cdot \hat{p}) S^+(P) + (\gamma^0 + \vec{\gamma} \cdot \hat{p}) S^-(P) \right]
\]

\[
S^\pm_R(P) = \left. \frac{i}{p^0 + \omega_0^2 \left( 1 - \frac{p^0 + p}{2p} \ln \left( \frac{p^0 + p}{p^0 - p} \right) \right)} \right|_{p^0 = p^0 + i\epsilon}
\]
Fermionic sum rules

\[ (2\pi)^3 \frac{d\Gamma}{d^3 k_{\text{soft}}} \propto \int \frac{dp^+ d^2 p_{\perp}}{(2\pi)^3} \text{Tr} \left[ \gamma^- (S_R(P) - S_A(P)) \right]_{p^- = 0} \]
Fermionic sum rules

\[ (2\pi)^3 \frac{d\Gamma}{d^3 k_{\text{soft}}} \propto \int \frac{dp^+ d^2 p_\perp}{(2\pi)^3} \text{Tr} \left[ \gamma^- (S_R(P) - S_A(P)) \right]_{p^- = 0} \]

- A retarded propagator is an analytic function of Q in the upper half-plane not just in the frequency, but in any time-like or light-like variable.
Fermionic sum rules

\[
(2\pi)^3 \frac{d\Gamma\gamma}{d^3 k_{\text{soft}}} \propto \int \frac{dp^+ d^2p_\perp}{(2\pi)^3} \text{Tr} \left[ \gamma^- (S_R(P) - S_A(P)) \right]_{p^- = 0}
\]

- A retarded propagator is an analytic function of Q in the upper half-plane not just in the frequency, but in any time-like or light-like variable.
Fermionic sum rules

\[(2\pi)^3 \frac{d\Gamma\gamma}{d^3 k_{soft}} \propto \int \frac{dp^+ d^2 p_\perp}{(2\pi)^3} \text{Tr} \left[ \gamma^- (S_R(P) - S_A(P)) \right]_{p^- = 0}\]

- A retarded propagator is an analytic function of Q in the upper half-plane not just in the frequency, but in any time-like or light-like variable
- Deform the contour away from the real axis
Fermionic sum rules

\[(2\pi)^3 \frac{d\Gamma}{d^3 k_{\text{soft}}} \propto \int \frac{dp^+ d^2 p_\perp}{(2\pi)^3} \text{Tr} \left[ \gamma^- \left( S_R(P) - S_A(P) \right) \right]_{p^-=0} \]

- Along the arcs at large complex \( p^+ \) the integrand has a very simple behavior

\[ \text{Tr} \left[ \gamma^- \left( S_R(P) - S_A(P) \right) \right]_{p^-=0} = \frac{i}{p^+} \frac{m_\infty^2}{p_\perp^2 + m_\infty^2} + O \left( \frac{1}{(p^+)^2} \right) \]
Fermionic sum rules

\[
(2\pi)^3 \frac{d\Gamma}{d^3 k_{\text{soft}}} \propto \int \frac{dp^+ \, d^2p_\perp}{(2\pi)^3} \text{Tr} \left[ \gamma^- (S_R(P) - S_A(P)) \right]_{p^- = 0}
\]

- Along the arcs at large complex \( p^+ \) the integrand has a very simple behavior

\[
\text{Tr} \left[ \gamma^- (S_R(P) - S_A(P)) \right]_{p^- = 0} = \frac{i}{p^+} \frac{m^2_\infty}{p_\perp^2 + m^2_\infty} + O \left( \frac{1}{(p^+)^2} \right)
\]

- The integral then gives simply

\[
(2\pi)^3 \frac{d\Gamma}{d^3 k_{\text{soft}}} \propto \int \frac{d^2p_\perp}{(2\pi)^2} \frac{m^2_\infty}{p_\perp^2 + m^2_\infty}
\]
Fermionic sum rules

\[(2\pi)^3 \frac{d\Gamma}{d^3k_{\text{soft}}} \propto \int \frac{dp^+ d^2p_\perp}{(2\pi)^3} \text{Tr} \left[ \gamma^- (S_R(P) - S_A(P)) \right]_{p^- = 0}\]

- Along the arcs at large complex \(p^+\) the integrand has a very simple behavior

\[
\text{Tr} \left[ \gamma^- (S_R(P) - S_A(P)) \right]_{p^- = 0} = \frac{i}{p^+} \frac{m_\infty^2}{p_\perp^2 + m_\infty^2} + \mathcal{O}\left(\frac{1}{(p^+)^2}\right)
\]

- The integral then gives simply

\[(2\pi)^3 \frac{d\Gamma}{d^3k_{\text{soft}}} \propto \int \frac{d^2p_\perp}{(2\pi)^2} \frac{m_\infty^2}{p_\perp^2 + m_\infty^2}\]

- The \(p_\perp\) integral is UV-log divergent, giving the LO UV-divergence that cancels the IR divergence at the hard scale, now analytically

Independently obtained by Besak Bödeker \textbf{JCAP1203} (2012)
The NLO soft region

At NLO one can use the KMS relations and the ra basis to write the diagrams in terms of fully retarded and fully advanced functions of $P$. The hard only depend on $p^-$. The contour deformations are then again possible and the diagrams can be expanded for large complex $p^+$. On general grounds we expect

\[
(2\pi)^3 \left. \frac{d\delta \Gamma}{d^3 k} \right|_{\text{soft}} \propto \int \frac{dp^+ d^2 p_\perp}{(2\pi)^3} \left[ C_0 \left( \frac{1}{p^+} \right)^0 + C_1 \left( \frac{1}{p^+} \right)^1 + \ldots \right]
\]
The soft region

The $(1/p^+)^0$ term has to be *exactly* the subtraction term we have mentioned before in the collinear region, to cancel the cutoff dependence. Confirmed by explicit calculation.

- At order $1/p^+$ we had the LO result. We can expect

\[
\frac{m_{\infty}^2}{p_\perp^2 + m_{\infty}^2} \rightarrow \frac{m_{\infty}^2 + \delta m_{\infty}^2}{p_\perp^2 + m_{\infty}^2 + \delta m_{\infty}^2} = \left( \frac{m_{\infty}^2}{p_\perp^2 + m_{\infty}^2} + \frac{\delta m_{\infty}^2 p_\perp^2}{(p_\perp^2 + m_{\infty}^2)^2} + \mathcal{O}(g^2) \right)
\]

The explicit calculation finds just this contribution.

- The contribution from HTL vertices goes like $(1/p^+)^2$ or smaller on the arcs.

\[
\sim \frac{1}{(p^+)^2}
\]
The semi-collinear region

- Seemingly different processes boiling down to wider-angle radiation

\[ K + P \]
\[ K \]
\[ P + Q \]
\[ Q \]

\[ P \text{ semi-collinear} \]
\[ Q \text{ soft} \]

\[ \sqrt{g} \]

Q soft plasmon, timelike

Q soft cut, spacelike

\[ g \]
The semi-collinear region

- Seemingly different processes boiling down to wider-angle radiation

Evaluation: introduce "modified $\hat{q}$" that keep tracks of the changes in the small light-cone component $p^-$ of the quarks

"standard" 

$$\frac{\hat{q}}{g^2 C_R} \equiv \frac{1}{g^2 C_R} \int \frac{d^2q_\perp}{(2\pi)^2} q_\perp^2 C(q_\perp) \propto \int d^4Q \langle F^{+\mu}(Q)F^{+\mu}_{\mu}(-Q)\rangle_{q^-=0}$$

"modified" 

$$\frac{\hat{q}(\delta E)}{g^2 C_R} \propto \int d^4Q \langle F^{+\mu}(Q)F^{+\mu}_{\mu}(-Q)\rangle_{q^-=\delta E}$$

The "modified $\hat{q}$" can also be evaluated in EQCD
NLO at large $K^2$

- Before showing any results, let us look at the large-$M$ region

$$k^0 + k \sim T \quad k^0 - k \sim T$$

- As we have seen, the Born term is proportional to $K^2$, which is now large ($\sim T^2$), so that the Born term is a well defined LO ($\alpha_{\text{EM}} g^0$)
NLO at large $K^2$

- At NLO, HTL and LPM resummations are no longer necessary.
- Very complicated two-loop integrals with intricate kinematics. Interplay of real and virtual corrections with cancellations of IR divergences.

Laine JHEP1305, JHEP1311 (2013)
Matching small and large $K^2$

- The large-$M$ calculation diverges logarithmically for $M \to 0$.

- The small-$M$ calculation extrapolates for large $M$ to $q \propto K^2 + T^2$, in violation of OPE results forbidding a $T^2$ term Caron-Huot PRD79 (2009).

- A procedure has been devised to combine the two calculations. In a nutshell,
  \[
  \rho_{\text{merge}} = \rho_{\text{large\,}M} + \rho_{\text{LPM}} - \rho_{\text{LPM}} K^2 \gg T^2
  \]
  where $q_{\text{LPM}}$ is the LO collinear part. NLO can be added easily.
• Full lines: JG Moore, valid at small $K^2$, does not include Laine (large $M$)
Dashed lines: Ghisoiu Laine, valid at large $K^2$

• At $\alpha_s=0.3$ the transition at the light cone is smooth
• Full lines: JG Moore, valid at small $K^2$, does not include Laine (large $M$)
  Dashed lines: Ghisoiu Laine, valid at large $K^2$

• At $\alpha_s=0.3$ the transition at the light cone is smooth
Come visit our website


  
  - at finite \( k \): Ghisoiu Laine plus JG Moore plus vacuum corrections to the Born term
  
  - at zero \( k \): transport peak from Moore Robert (2006), \( k^0 > \pi T \), NLO thermal from Aurenche Altherr (1989), vacuum corrections to the Born term. Missing reliable pQCD input in the intermediate region

  - JG Laine
$T = 1.1T_c \quad k = 2\pi T$

Born

“Vacuum”

Large M+LO soft M

Large M+NLO soft M
Towards a lattice comparison
EM probes and the lattice

- What is measured directly is the Euclidean correlator

\[ G_E(\tau, k) = \int d^3 x J_\mu(\tau, x) J_\mu(0, 0) e^{i k \cdot x} \]
What is measured directly is the Euclidean correlator
\[ G_E(\tau, k) = \int d^3x J_\mu(\tau, x) J_\mu(0, 0) e^{i k \cdot x} \]

Analytical continuation
\[ G_E(\tau, k) = G^< (i\tau, k) \]

\[ G_E(\tau, k) = \int_0^\infty \frac{d k^0}{2\pi} \rho_V(k^0, k) \frac{\cosh(k^0(\tau - 1/2T))}{\sinh(k^0/2T)} \]
EM probes and the lattice

- What is measured directly is the Euclidean correlator
  \[ G_E(\tau, k) = \int d^3 x J_\mu(\tau, x) J_\mu(0, 0) e^{i k \cdot x} \]

- Analytical continuation \[ G_E(\tau, k) = G^< (i\tau, k) \]
  \[ G_E(\tau, k) = \int_0^\infty \frac{dk^0}{2\pi} \rho_V(k^0, k) \frac{\cosh(k^0(\tau - 1/2T))}{\sinh(k^0/2T)} \]

- It contains a lot of info (full spectral function), but hidden in the convolution. Inversion tricky, discrete dataset with errors
EM probes and the lattice

- Work in progress JG Kaczmarek Laine Meyer...
  - Continuum-extrapolated lattice data at finite momentum
  - Comparison with the pQCD Euclidean correlator
    \[ G_E(\tau, k) = \int_0^\infty \frac{dk^0}{2\pi} \rho_V(k^0, k) \frac{\cosh(k^0(\tau - 1/2T))}{\sinh(k^0/2T)} \]
  - Try to make best use of pQCD input to extract the spectral function
At finite momentum

- If $k>0$ spf describes DIS ($k^0<k$), photons ($k^0=k$) and dileptons ($k^0>k$).

![Born spectral function graph]

- $\rho_{JJ}(\omega,k=5)/\omega T$
- $\tau = 0.1/T$
- $\tau = 0.4/T$
At finite momentum

- Comparison with the pQCD Euclidean correlator

\[
\frac{G_V}{G_V^{\text{free}}} \text{ at } p = k \times 2\pi\left(\frac{7}{24}\right) \quad T = 1.2T_c
\]

JG Kaczmarek Laine Meyer preliminary
At finite momentum

• The perturbative data overshoots the lattice data. Too much support at low frequency?

• Try a fitting Ansatz: perturbative, thermal spf above $M \sim \pi T$. Fifth-degree polynomial in $k^0$, with odd powers only, below $M \sim \pi T$ (three coefficients)

• Constrain two coefficients by requiring smoothness in spf and first derivative at the matching point. Fit the remaining coefficient to lattice data
At finite momentum

- The perturbative data overshoots the lattice data.
- Too much support at low frequency?
- Try a fitting Ansatz: perturbative, thermal spf above $M \sim T$.
  - Fifth-degree polynomial in $k_0$, with odd powers only, below $M \sim T$ (three coefficients)
- Constrain two coefficients by requiring smoothness in spf and first derivative at the matching point. Fit the remaining coefficient to lattice data.

- Results qualitatively similar at $T=1.2T_c$
- Lattice continuum extrapolation reliable only from $\tau T > 0.18$
- Matching point at $k^0 = k + 1.5T$

JG Kaczmarek Laine Meyer preliminary.
At finite momentum

- Results qualitatively similar at $T=1.2T_c$
- The fit has a good $\chi^2$, which also has a local minimum for $M \sim \pi T$ and the spf at the photon point is stable against varying the matching point
- Interesting: if $k$ small enough we could probe the collective region (diffusion/conductivity)

Figure 2: We show $\chi^2$/d.o.f. (top) and $\rho_V T/(2k\chi)$ (bottom) as a function of the matching point $\omega_0$. A local minimum of $\chi^2$/d.o.f. is generally found close to the point where $\omega_0 = \sqrt{k^2 + (\pi T)^2}$.

To be added: error bars, in particular

- How to handle the error of $\chi^2$?
- Dependence of the results on the order of the polynomial ($n_{\text{max}} \geq 1$)?
- Bootstrap sample?
- Error on the pQCD side from uncertainty in $T/T_c$ or RG scale choice?

6. Conclusions
We have shown how a combination of lattice and perturbative results allows us to obtain non-trivial information about the vector channel spectral function close to the photon point. The observed relative stability of the perturbative result is consistent with the smallness of the NLO correction [11, 17], as well as on indirect checks concerning the convergence of the weak-coupling expansion for light-cone observables, based on measuring screening masses at non-zero Matsubara frequencies [35]. We have also demonstrated that, even though not constrained a priori, the fit result reproduces some qualitative features expected from the soft domain, namely a reduced (and possibly even negative) spectral weight in the spacelike domain. In addition, as has been illustrated in fig. 4, measurements at non-zero momentum offer an alternative way to...
At finite momentum

- At the photon point modest changes from pQCD expectations (below 20% except perhaps at the smallest ks, also at 1.2 $T_c$). Good for pheno!
- AdS/CFT curve adjusted to asymptote to the bare QCD result (extra symmetries make $T=0$ curve coupling-independent)

Conclusions

• NLO calculations for dileptons are now available over a wide range of invariant masses (at finite $k$)

• In both cases convergence seems reasonable. At small $K^2$ transition to photon is smooth

• A collection of the best available data has been prepared and is ready for use by pheno/lattice practitioners

• Comparison and collaboration with lattice groups is ongoing. A simple Ansatz gives a good fit and seems to further suggest stability (at the tens of % level) of the pQCD rates
Backup
Summary

• **LO rate**

\[
(2\pi)^3 \frac{d\Gamma}{d^3k} \bigg|_{\text{LO}} = A(k) \left[ \log \frac{T}{m_\infty} + C_{2\rightarrow2}(k) + C_{\text{coll}}(k) \right]
\]

\[
A(k) = \alpha_{\text{EM}} g^2 \frac{C_F T^2 n_F(k)}{2k} \sum_f Q_f^2 d_f
\]

• **NLO correction**

\[
(2\pi)^3 \frac{d\delta\Gamma}{d^3k} \bigg|_{\text{NLO}} = A(k) \left[ \frac{\delta m^2}{m_\infty^2} \log \frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m^2}{m_\infty^2} C_{\text{soft+sc}}(k) + \frac{\delta m^2}{m_\infty^2} C_{\text{coll}}(k) + \frac{g^2 C_A T}{m_D} C_{\text{coll}}(k) \right]
\]

• **Fits available in the paper**

JG Hong Kurkela Lu Moore Teaney *JHEP0513 (2013)*
\[
(2\pi)^3 \left| \frac{d\delta \Gamma}{d^3k} \right|_{NLO} = A(k) \delta C_{\text{NLO}}(k) = \begin{cases} 
\frac{\delta m^2}{m^2_\infty} \log \frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m^2}{m^2_\infty} C_{\text{soft+sc}}(k) + \frac{\delta m^2}{m^2_\infty} C_{\text{coll}}^{\delta m}(k) + \frac{g^2 C_A T}{m_D} C_{\text{coll}}^{\delta c}(k) 
\end{cases}
\]

\[\alpha_S = 0.3\]

\[\begin{align*}
C_{\text{LO}} + \delta C_{\text{coll}} \\
C_{\text{LO} + \text{NLO}} \\
C_{\text{LO}} \\
C_{\text{LO}} + \delta C_{\text{soft+sc}}
\end{align*}\]
\[(2\pi)^3 \frac{d\delta \Gamma}{d^3k}\bigg|_{\text{NLO}} = A(k) \left[ \frac{\delta m^2}{m_\infty^2} \log \frac{\sqrt{2} T m_D}{m_\infty} + \frac{\delta m^2}{m_\infty^2} C_{\text{soft+sc}}(k) + \frac{\delta m^2}{m_\infty^2} C_{\text{coll}}^m(k) + \frac{g^2 C_A T}{m_D} C_{\text{coll}}^{\delta c}(k) \right] \]

\[\delta C_{\text{NLO}}(k) = \delta C_{\text{soft+sc}}(k) + \delta C_{\text{coll}}(k)\]

\[\alpha_s = 0.3\]

\[\frac{C(k)}{C_{\text{LO}}} + \delta C_{\text{coll}}\]

\[C_{\text{LO}}\]

\[C_{\text{LO+NLO}}\]

\[C_{\text{LO+} \delta C_{\text{soft+sc}}}\]

\[\alpha_s = 0.3\] for the momentum range (Fig. 18). At large momentum, the collinear correction is nonvanishing in an Abelian theory. Given those definitions, it then follows that...
than in the previous case. and the magnitude of the two largely canceling contributions is also significantly smaller

For the smaller coupling constant the NLO correction is always negative and rather flat,

large cancellations we observe are rather accidental, and one should thus consider the

included, with the analogous notation for the LO + soft+sc curve. The di

The dashed curve labeled LO+coll shows the ratio of rates when only the collinear correction is

respectively of an “uncertainty estimate” of the NLO calculation.

uncertainty estimate for the NLO calculation. (b) The same as (a) for larger

C

Figure 19

The dashed curves provides a uncertainty estimate for the NLO calculation. (b) The same as (a)

In Fig.

and

0.5

1.5

2

3

4

5

0

2

1.5

1

0.5

0

2

4

6

8

10

12

14

k/T

(LO + NLO)/LO

(LO + coll)/LO

(LO + soft+sc)/LO

\(\alpha_s = 0.30\)

\(0\)

\(2\)

\(4\)

\(6\)

\(8\)

\(10\)

\(12\)

\(14\)

k/T

\(\alpha_s = 0.30\)

\(0\)

\(10\)

\(20\)

\(30\)

\(40\)

\(50\)

\(60\)

\(70\)

\(80\)

\(90\)
impact parameter space and the resulting tribute here. We have dealt with the collinear region in Sec. collinear regions receive
regions are divided. However, this dependence cancels in the sum. At NLO the soft and hard and the soft regions have logarithmic sensitivity to the details of how the kinematical regimes — the hard, soft and collinear regions. The contributions arising from the quark-gluon plasma. The contributions to the LO rate can be divided into distinct kine-
We have computed the photon production rate to NLO of an equilibrated, weakly-coupled

Conclusions

(Fig. 21

curves
large cancellations we observe are rather accidental, and one should thus consider the
The dashed curve labeled LO+coll shows the ratio of rates when only the collinear correction is
leading order rate (LO), a collinear correction (coll), and a soft+semi-collinear correction (soft+sc).

Figure 20

(LO+NLO) / LO

Figure 21

αs=0.05

k/T

C(k)

CLO

CLO+δCcoll

CLO+δCsoft+sc

CLO+NLO

Lo+coll

LO

LO+soft+sc

(LO+NLO)

(LO+NLO)

LO+NLO

αs=0.05

k/T

(LO+NLO) / LO

(LO+NLO) / LO

(LO+NLO) / LO

(LO+NLO) / LO

αs=0.05

k/T