Astrophysical Haloscopes

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based on GS, arXiv:1708.08908, to appear in PRD

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Universität Hamburg
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ALP-photon Conversion in Structured Magnetic Fields

fundamental coupling:

$$\frac{\alpha_{\rm em}}{8\pi}\frac{C_{a\gamma}}{f_a}\,a\,F_{\mu\nu}\tilde{F}^{\mu\nu}=\frac{e^2}{32\pi^2}\frac{C_{a\gamma}}{f_a}\,a\,F_{\mu\nu}\tilde{F}^{\mu\nu}=\frac{\alpha_{\rm em}}{8\pi}\frac{C_{a\gamma}}{f_a}\,a\,F_{\mu\nu}\tilde{F}^{\mu\nu}=\frac{g_{a\gamma}}{4}\,a\,F_{\mu\nu}\tilde{F}^{\mu\nu}\,, \tag{4}$$

where $\alpha_{\rm em}=e^2/(4\pi\epsilon_0)$ and

$$g_{a\gamma}\equiv\frac{\alpha_{\rm em}C_{a\gamma}}{2\pi f_a}\,. \tag{5}$$

Energy-momentum conservation: quantities for ALP, photon and magnetic field carry subscript a, γ, or none, respectively:

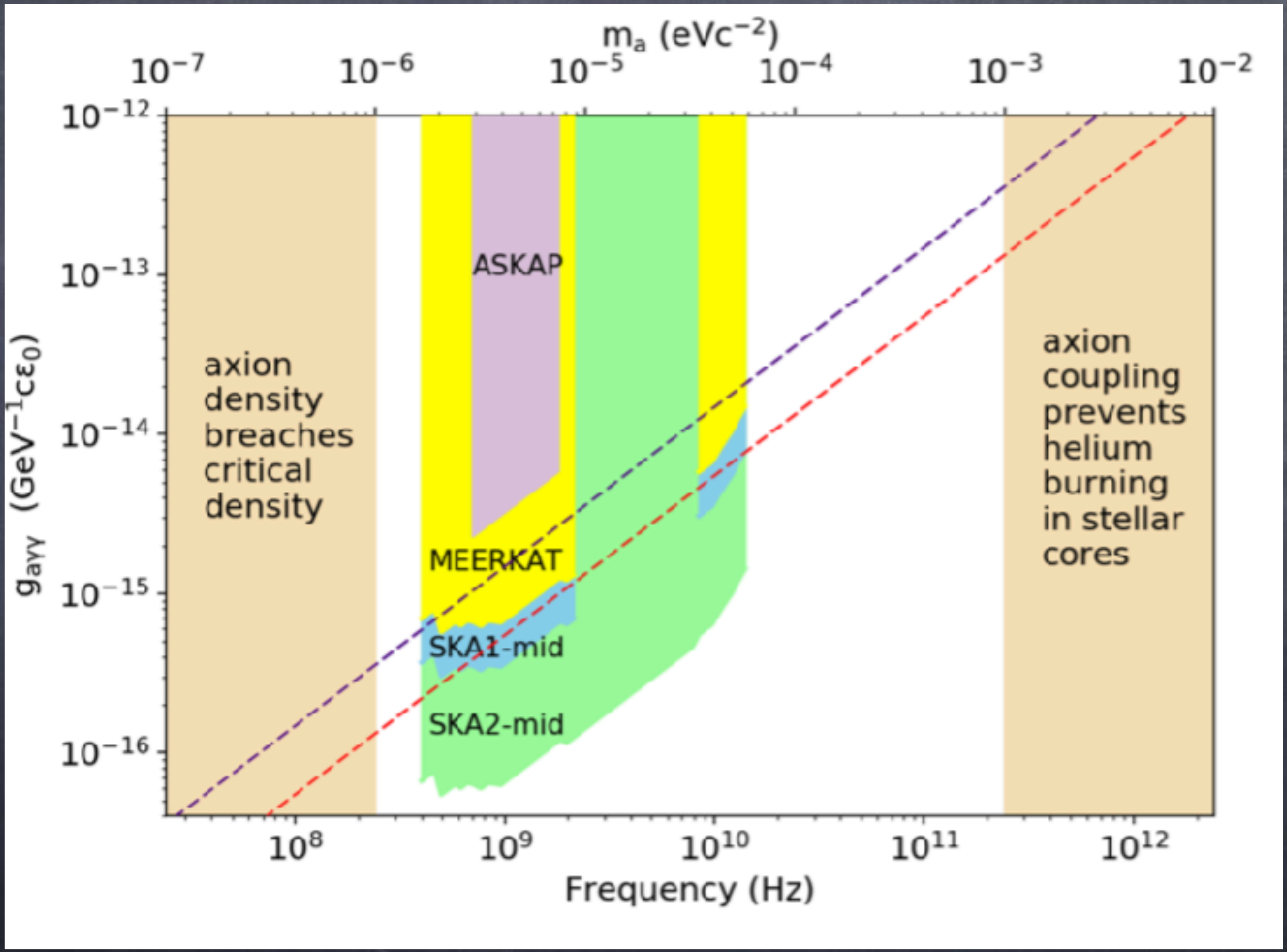
$$E_a=(m_a^2+\mathbf{k}_a^2)^{1/2}=\omega_\gamma-\omega=(\omega_{\rm pl}^2+\mathbf{k}_\gamma^2)^{1/2}-\omega\,,\quad \mathbf{k}_a=\mathbf{k}_\gamma-\mathbf{k}\,,$$

where the plasma frequency is given by:

$$\omega_{\rm pl}=\left(\frac{e^2n_e}{\epsilon_0 m_e}\right)^{1/2}\simeq 5.6\times10^4\left(\frac{n_e}{\rm cm^{-3}}\right)^{1/2}{\rm rad\,s^{-1}}\,.$$

propagation of converted photons requires $m_a \gg \omega_{\rm pl}$:

recently K. Kelley and P. J. Quinn, Astrophys. J. 845, 1 (2017) [arXiv:1708.01399] pointed out the possibility to search for ALP dark matter with radio telescopes; they used standard magnetic field estimates but assumed most of the power is on meter scales which is unlikely.



For non-relativistic (dark matter) ALPs, $k_a \ll m_a$, k_γ one can then show that the ALP-photon conversion rate can be written as

$$R_{a\to\gamma} = \frac{\pi\epsilon_0}{2} g_{a\gamma}^2 n_a \int \frac{d\omega}{T} d^3\mathbf{k}_\gamma \delta(\omega + E_a - \omega_\gamma) \sum_\lambda |\mathbf{B}(\omega, \mathbf{k}_\gamma - \mathbf{k}_a) \cdot \epsilon_\lambda(\mathbf{k}_\gamma)|^2$$
$$= \frac{\pi\epsilon_0}{2} g_{a\gamma}^2 n_a \frac{1}{T} \int d^3\mathbf{k}_\gamma \sum_\lambda |\mathbf{B}(\omega_\gamma - E_a, \mathbf{k}_\gamma - \mathbf{k}_a) \cdot \epsilon_\lambda(\mathbf{k}_\gamma)|^2 \ ,$$

where ϵ_λ is the photon polarisation and T is the integration time. For MHD waves with dispersion relation $\omega = v_m k$, $v_m \ll 1$, this gives rise to a narrow line of relative width

$$\Delta \equiv \frac{\Delta k_\gamma}{k_\gamma} \simeq v_a^2/2 + v_m + \Delta v \lesssim 10^{-3} \ ,$$

where $v_a \sim 10^{-3}$ = characteristic ALP velocity, Δv = velocity dispersion within the object and $v_m \lesssim$ Alfven velocity $\sim 10^{-5}$ (B/μG)(1 cm^{-3}/n)$^{1/2}$
In the limit $\omega \ll k$ (almost static magnetic fields) this gives

$$R_{a\to\gamma} = \frac{\pi\epsilon_0}{2} g_{a\gamma}^2 n_a \int d^3\mathbf{k}_\gamma \delta(k_\gamma - E_a) \sum_\lambda |\mathbf{B}(\mathbf{k}_\gamma - \mathbf{k}_a) \cdot \epsilon_\lambda(\mathbf{k}_\gamma)|^2 \ .$$

More compactly this can be written in terms of the magnetic field (static) power spectrum defined as

$$\rho_m = \frac{1}{2\mu_0 V}\int d^3\mathbf{r}|\mathbf{B}(\mathbf{r})|^2 = \frac{1}{2\mu_0 V}\int d^3\mathbf{k}|\mathbf{B}(\mathbf{k})|^2 = \int d\ln k\,\rho_m(k)\,,$$

Using $|\mathbf{k}_\gamma - \mathbf{k}_a| \sim k_\gamma \sim m_a$ and assuming a **homogeneous ALP distribution** with total mass $M_a = n_a m_a V$ this gives

$$R_{a\to\gamma} \simeq \pi g_{a\gamma}^2 \frac{M_a}{m_a^2}\rho_m(m_a)\,,$$

Integration over the line of sight dl this results in a specific intensity per solid angle [Jansky per steradian where 1 Jy = 10^{-26} W/(cm^2 Hz) = 10^{-23} erg/(cm^2 s Hz)]

$$I \simeq \pi \frac{g_{a\gamma}^2}{m_a^2}\frac{1}{\Delta}\int_{\mathrm{l.o.s.}} dl\,\rho_a(l)\,\rho_m(m_a,l)\,,$$

For a source at distance d containing total ALP mass M_a one similarly gets the **total flux density (Jansky)**

$$S \simeq \frac{\pi}{4d^2}\frac{g_{a\gamma}^2}{m_a^2}\frac{1}{\Delta}\int d^3\mathbf{r}\rho_a(\mathbf{r})\rho_m(m_a,\mathbf{r}) \simeq \frac{\pi}{4d^2}\frac{g_{a\gamma}^2}{m_a^2}\frac{1}{\Delta}M_a\rho_m(m_a)\,,$$

For a source radius r_s this corresponds to the specific intensity $I=S/\Omega_s$, which is independent of the distance:

$$I \simeq \frac{1}{4r_s^2}\frac{g_{a\gamma}^2}{m_a^2}\frac{1}{\Delta}M_a\rho_m(m_a)\,,$$

Application to Astrophysical Sources

radio photon frequency and wavenumber:

$$\nu = \omega_\gamma/(2\pi) = 242 \left(\frac{m_a}{\mu\mathrm{eV}}\right) \mathrm{MHz}\,, \quad \frac{1}{k} = 20 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-1} \mathrm{cm}\,.$$

Ansatz for the magnetic field power spectrum:

$$\rho_m(k) = \frac{B^2}{2\mu_0} f(k)\,,$$

where f(k) is the fraction of the total power at k which is often written as a power law $f(k) \sim (kl_c)^n$ with l_c the coherence length, e.g. n=-2/3 for Kolmogorov turbulence, extending between l_c and the resistive scale

$$\lambda_r \simeq 0.9 \left(\frac{10^6\,\mathrm{K}}{T}\right)^{3/2} \left(\frac{B_0}{\mu\mathrm{G}}\right)^{-1} \left(\frac{n}{\mathrm{cm}^{-3}}\right)^{1/2} \mathrm{cm}\,.$$

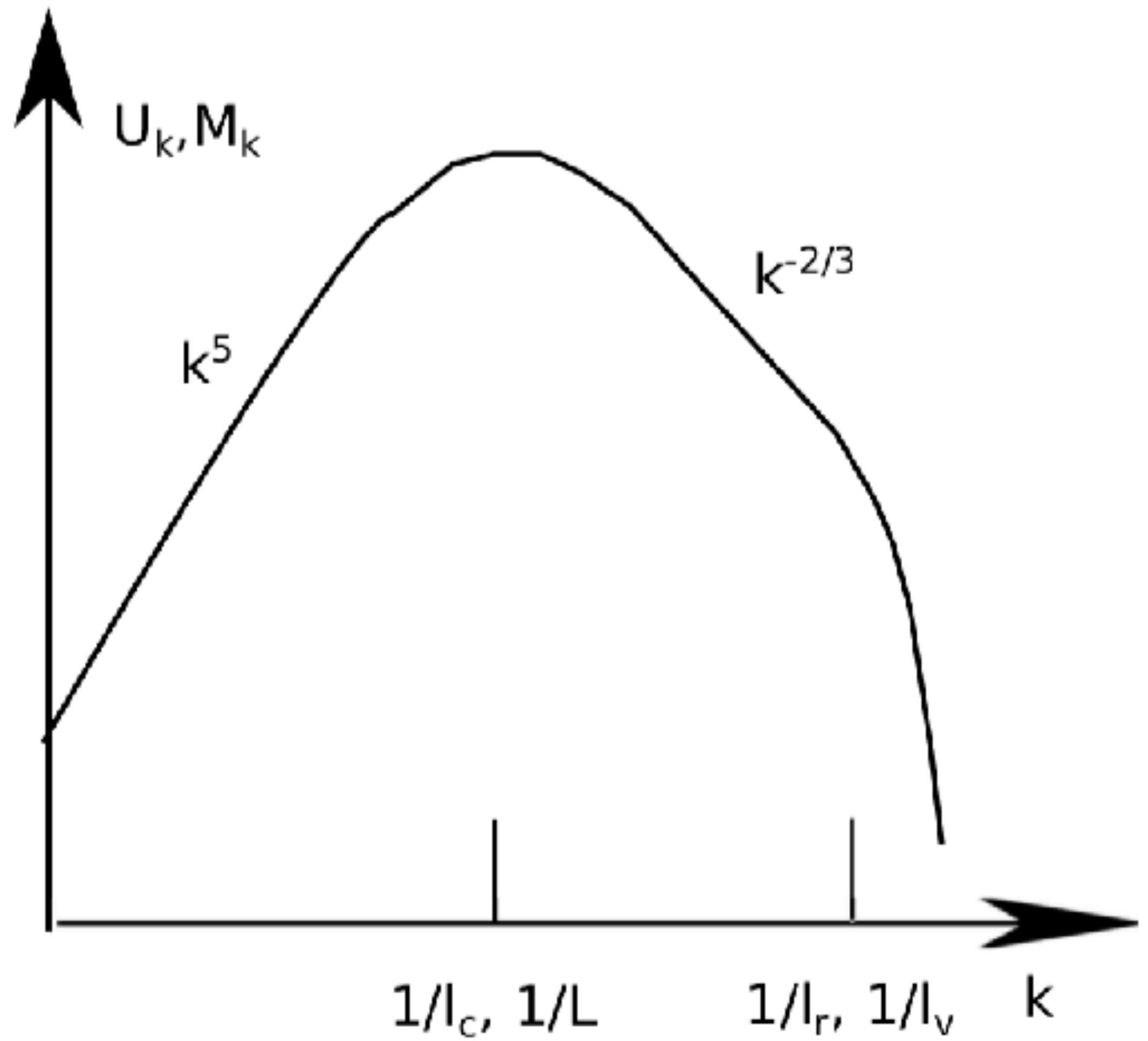


Fig. 3.4 A typical power spectrum of the magnetic field or kinetic fluid flow energy density per logarithm of wavenumber k, M_k and U_k, respectively, on logarithmic scales. In the inertial range between coherence length l_c of the magnetic field which is comparable to the energetically dominant eddy length L, and the resistive and viscous length scale l_r and l_v, respectively, a universal Kolmogorov turbulence spectrum $M_k, U_k \propto k^{-2/3}$ is indicated. At length scales $l \ll l_r, l_v$ the power spectrum is usually exponentially suppressed due to dissipation. At length scales $l \gg l_c, L$ the universal slope $M_k, U_k \propto k^5$ is indicated.

G. Sigl, book "Astroparticle Physics: Theory and Phenomenology", Atlantis Press/Springer 2016

With these ansätze the conversion rate per ALP becomes

$$\frac{1}{\tau_a} \simeq \pi g_{a\gamma}^2 \frac{1}{m_a} \rho_m(m_a) \simeq 9.7 \times 10^{-29} \left(g_{a\gamma} 10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-1} \left(\frac{B}{\mathrm{G}}\right)^2 f(m_a)\,\mathrm{s}^{-1},$$

Often one defines a brightness temperature by using the relation between temperature and specific intensity for black body radiation in the Rayleigh-Jeans limit:

$$T_b(\nu) \equiv \frac{c_0^2 I}{2\nu^2},$$

which as I is independent of the distance. For our case ν = ma/(2π) one gets

$$T_b(m_a) \equiv 2\pi^2 c_0^2 I/m_a^2 = 0.56 \left(\frac{I}{\mathrm{Jy/sr}}\right) \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-2} \mathrm{mK}.$$

For the galactic diffuse emission with galactic magnetic field amplitude B one gets

$$I \simeq 1.8\,\left(g_{a\gamma}10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-2} \left(\frac{10^{-3}}{\Delta}\right) \left(\frac{\rho_a}{0.3\,\mathrm{GeV\,cm^{-3}}}\right) \left(\frac{L}{8\,\mathrm{kpc}}\right) \times \left(\frac{B}{5\,\mu\mathrm{G}}\right)^2 f(m_a)\,\frac{\mathrm{mJy}}{\mathrm{sr}}\,,$$

where L is the characteristic length scale of the Milky Way. Advantage: **Insensitive to ALP inhomogeneities.**

For a discrete object at distance d containing total ALP mass M_a with characteristic magnetic field B the total flux density will be

$$S \simeq 2.8 \times 10^{-11}\,\left(g_{a\gamma}10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-2} \left(\frac{10^{-3}}{\Delta}\right) \left(\frac{M_a}{10^{-10}\,M_\odot}\right) \left(\frac{d}{\mathrm{kpc}}\right)^{-2} \left(\frac{B}{\mathrm{G}}\right)^2 f(m_a)\,\mathrm{Jy}\,.$$

corresponding to a brightness temperature of

$$T_b \simeq 5\,\left(g_{a\gamma}10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-4} \left(\frac{10^{-3}}{\Delta}\right) \left(\frac{M_a}{10^{-10}\,M_\odot}\right) \left(\frac{r_s}{\mathrm{pc}}\right)^{-2} \left(\frac{B}{\mathrm{G}}\right)^2 f(m_a)\,\mathrm{nK}\,.$$

Supernova remnants and magnetized stellar winds have r_s ~ pc, B ~ mG, d ~ kpc: Radius r_s of termination shock can be estimated by equating mass of interstellar medium swept up with ejecta mass:

$$r_s \sim \left(\frac{3M_e}{4\pi m_N n_0}\right)^{1/3} \simeq 2.1 \left(\frac{M_e}{M_\odot}\right)^{1/3} \left(\frac{1\,\mathrm{cm}^{-3}}{n_0}\right)^{1/3} \mathrm{pc}\,,$$

Magnetic field can be estimated from equipartition of magnetic and kinetic wind energy:

$$B(r_s) \sim \left(\frac{3\mu_0 M_e}{4\pi r_s^3}\right)^{1/2} v_w \simeq (\mu_0 m_N n_0)^{1/2} v_w \simeq 1.9\times 10^{-3} \left(\frac{n_0}{1\,\mathrm{cm}^{-3}}\right)^{1/2} \left(\frac{v_w}{10^{-2}}\right) \mathrm{G}\,,$$

This gives:

$$S \simeq 2.1\times 10^{-8}\, \left(g_{a\gamma} 10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-2} \left(\frac{10^{-3}}{\Delta}\right) \left(\frac{d}{2\,\mathrm{kpc}}\right)^{-2} \left(\frac{B}{10^{-3}\,\mathrm{G}}\right)^2 f(m_a)\,\mathrm{Jy}\,,$$

$$T_b \simeq 3.8\, \left(g_{a\gamma} 10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-4} \left(\frac{10^{-3}}{\Delta}\right) \left(\frac{B}{10^{-3}\,\mathrm{G}}\right)^2 f(m_a)\,\mu\mathrm{K}\,.$$

Supernova remnants are too radio-loud [10^3 Jy, brightness temperature ~ 10^5 K], but stellar winds, e.g. Wolf-Rayet stars could be sufficiently quiet.

Magnetic Field Structure

coherence length l_c is often of order size of the object unless instabilities act on small scales:

Weibel instability would develop on the Debye length scale

$$\lambda_D = \frac{\bar{v}}{\omega_{\rm pl}} \simeq \left(\frac{\epsilon_0 T_e}{e^2 n_e}\right)^{1/2} \simeq 6.9 \times 10^3 \left(\frac{T_e}{10^6 {\rm K}}\right)^{1/2} \left(\frac{{\rm cm}^{-3}}{n_e}\right)^{1/2} {\rm cm}\,,$$

Bell instability can develop self-induced magnetic fields by accelerated cosmic rays and would develop on the gyro radius of accelerated cosmic ray ions

$$r_g \sim 3 \times 10^9 \left(\frac{10^{-3}{\rm G}}{B}\right) \left(\frac{p/Z}{\rm GeV}\right) {\rm cm}\,.$$

One needs MHD modes with $\omega = v_m\, k \ll k$ but their intensity in hot plasmas with magnetic fields coherent on the wavenumber scale m_a is hard to determine and needs to be worked out as a next step.

Example: MHD modes in cold, magnetised medium

Blandford+Thorne: Applications of classical physics (2003)

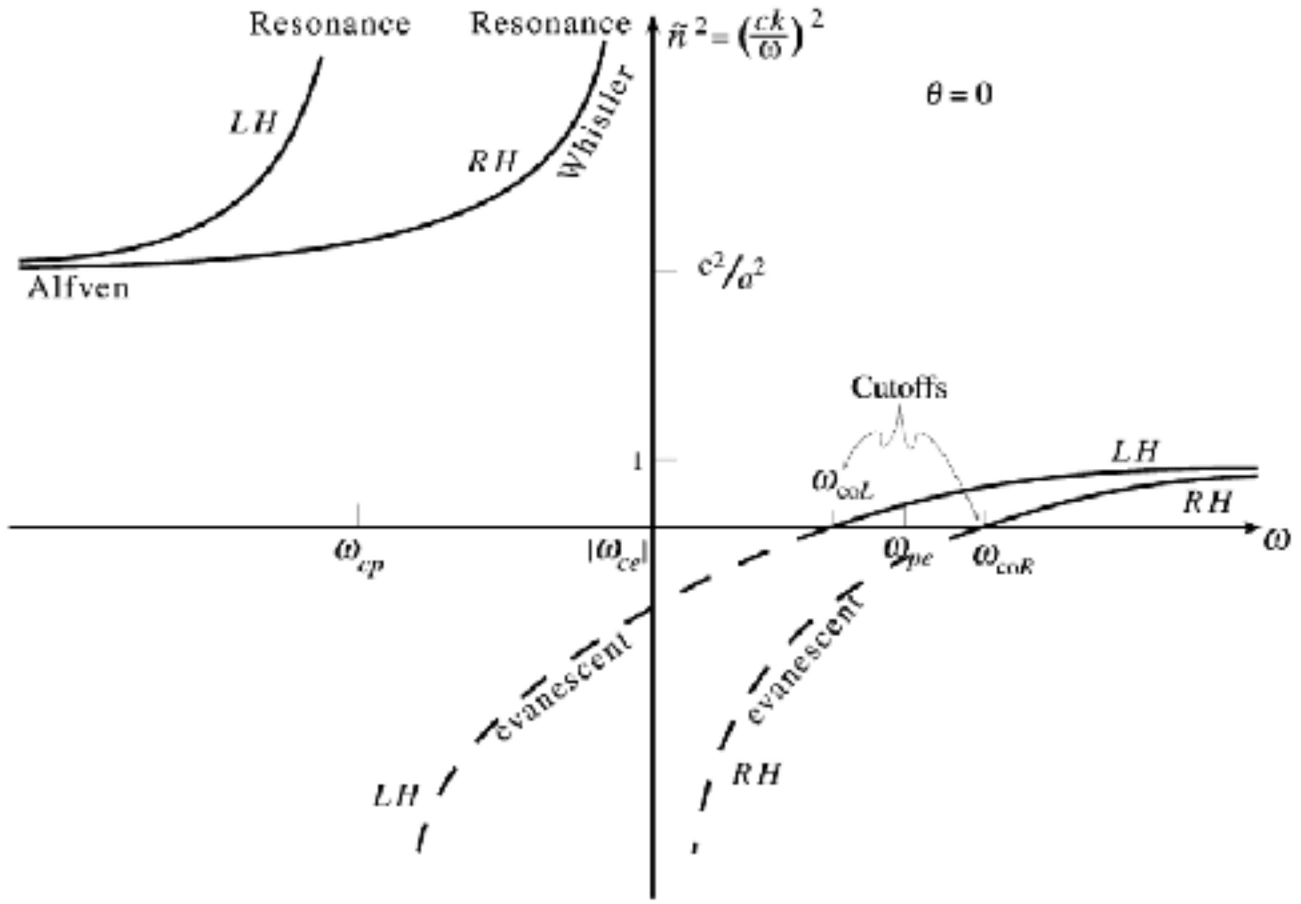


Fig. 20.3: Square of wave refractive index for circularly polarized waves propagating along the static magnetic field in a proton-electron plasma with $\omega_{pe} > \omega_{ce}$. (Remember, that we will regard both the electron and the proton cyclotron frequencies as positive numbers.) The angular frequency is plotted logarithmically in units of the modulus of the electron gyro frequency.

For $\omega_{ce} < \omega < \omega_{pl}$ there are Whistler modes that could give significant power. However, this requires $v_s \sim (T/m_N)^{1/2} < \omega/m_a \sim \omega_{g,e}/m_a \sim 10^{-5}$ (B/mG) which is hard to fulfil given that temperatures are T ~ eV-keV: need to better understand MHD modes in hot magnetised media

Detectability in Radio Telescopes

The effective solid angle of a single Gaussian beam is given by

$$\Omega_b \simeq \theta^2 \simeq \frac{1}{(l\nu)^2} = \frac{1}{A\nu^2}\,, \tag{36}$$

where θ is the angular radius of the beam, l is the effective length scale of the interferometer and $A = l^2$ its effective area. If a discrete source extends over several beams, the sensitivity in brightness temperature is increased by a factor $N_b^{1/2} = (\Omega_s/\Omega_b)^{1/2}$ relative to a single beam so that the minimal detectable brightness temperature is given by

$$T_{b,\text{min}} \simeq \frac{T_{b,\text{min0}}}{N_b^{1/2}} = T_{b,\text{min0}} \left(\frac{\Omega_b}{\Omega_s}\right)^{1/2}\,, \tag{37}$$

where $T_{b,\text{min0}}$ is the sensitivity for a single beam. In general one has

$$T_{b,\text{min0}} \simeq \frac{T_\text{noise}}{(Bt)^{1/2}}\,, \tag{38}$$

where T_noise is the effective noise temperature, resulting from system and sky temperature added in quadrature, B is the bandwidth and t is the observing time. One also often uses the antenna temperature induced by a total flux density S defined by

$$T_a \equiv \frac{AS}{2} = 0.36 \left(\frac{A}{10^3\,\text{m}^2}\right)\left(\frac{S}{\text{Jy}}\right)\text{K}\,. \tag{39}$$

Combining this with Eqs. (25) and (36) and the relation $S = I\Omega_s$ this shows that

$$\frac{T_a}{T_b} = N_b = \frac{\Omega_s}{\Omega_b}\,. \tag{40}$$

If the noise in one beam is again characterized by the temperature $T_{b,\min 0}$, the noise in N_b beams corresponds to $N_b^{1/2} T_{b,\min 0}$. Comparing this with the total signal temperature T_a again gives a brightness temperature sensitivity improvement by a factor $N_b^{1/2}$. Equivalently, the smallest detectable total source flux density can be expressed as

$$S_{\min} = N_b^{1/2} S_b \,, \tag{41}$$

where S_b is the minimal detectable flux density per beam. Since $S_{\min}$ and S_b are proportional to $T_{b,\min 0}$ which according to Eq. (38) is proportional to $1/t^{1/2}$, one often denotes the minimal detectable source flux density in units of $\mathrm{Jy\,hr^{-1/2}}$.

LOFAR HBA:

at ν = 140 MHz, beam size 5 arcsec, $\Omega_b = 2\times10^{-9}$ sr, sensitivity per beam $S_b \sim 10^{-4}$ Jy
for canonical case r_s = 2 pc, d = 2 kpc this gives $T_b \sim 2$ [d/(2 kpc)] K,
$S_{min} \sim 4\times10^{-3}$ [d/(2 kpc)] Jy; comparing with prediction gives sensitivity to

$$g_{a\gamma} \gtrsim 2.5 \times 10^{-12} \, [m_a/(0.58\,\mu\mathrm{eV})][\Delta/10^{-3}]^{1/2}[d/(2\,\mathrm{kpc})]^{1/2}\,\mathrm{GeV}^{-1}/f(m_a)$$

SKA-low:

frequency range (50-250) MHz, beam size ~ square degree, $\Omega_b = 3\times10^{-4}$ sr, larger than typical source, flux density sensitivity $S \sim 10^{-5}$ Jy/hr$^{1/2}$
this gives $T_b \sim 10^{-5}$ K/hr$^{1/2}$, corresponding to $T_{noise} \sim 10$ K and bandwidth B ~ 300 MHz; comparing with prediction for S gives sensitivity in one hour to

$$g_{a\gamma} \gtrsim 2 \times 10^{-13} \, [m_a/\mu\mathrm{eV}][\Delta/10^{-3}]^{1/2}[d/(2\,\mathrm{kpc})]^{1/2}\,\mathrm{GeV}^{-1}/f(m_a)$$

Conclusions

1.) Linelike radio emissions from dark matter-ALP conversion into photons in magnetic fields may be detectable with current and future radio telescopes such as LOFAR and SKA

2.) However, the most crucial (and least known) parameter is the magnetic field power on the ALP mass scale which is in the meter regime for μeV ALP masses. MHD modes in the presence of coherent magnetic fields would play an important role but their intensity is currently unclear.

Speculations

An "axion asteroid" would have ~ 10^{-13} M_{solar}, closest distance d ~ 100 AU ~ 5×10^{-7} kpc; for $g_{a\gamma}$ ~ 1/(10^{14} GeV), m_a ~ 10^{-6} eV and typical galactic field strength one gets 10^{-11} f(m_a) Jy at solid angles much smaller than the beam size. There would be many of the in the Milky Way, could this provide a measurable signature ? Probably comparable to diffuse flux

Full conversion (e.g. around neutron stars) gives

$$S_{\max} \simeq \frac{\rho_a}{m_a}\frac{v_a}{\Delta}\left(\frac{r_s}{d}\right)^2 \simeq 10^{-10}\left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-1}\left(\frac{r_s}{10^6\,\mathrm{cm}}\right)^2\left(\frac{d}{\mathrm{kpc}}\right)^{-2}\,\mathrm{Jy}\,,$$

see also M.S.Pshirkov, J.Exp.Theor.Phys. 108 (2009) 384 [arXiv:0711.1264] who obtained higher fluxes

This would be detectable out to ~ pc distances