SCALAR CHARGES FROM

2+1+1-FLAVOR LATTICE QCD

Huey-Wen Lin
Lattice QCD 101

§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories.

§ Physical observables are calculated from the path integral

\[ \langle 0 \vert O(\bar{\psi}, \psi, A) \vert 0 \rangle = \frac{1}{Z} \int DA \, D\bar{\psi} \, D\psi \, e^{iS(\bar{\psi}, \psi, A)} O (\bar{\psi}, \psi, A) \]

in Euclidean space.

☞ Quark mass parameter (described by \( m_\pi \))

☞ Impose a UV cutoff
discretize spacetime

☞ Impose an infrared cutoff
finite volume

§ Recover physical limit

\[ m_\pi \rightarrow m_\pi^{\text{phys}}, \, a \rightarrow 0, \, L \rightarrow \infty \]
**Nucleon Matrix Elements**

Lattice-QCD calculation of \( \langle N | \bar{q} \Gamma q | N \rangle \)

\[ \begin{align*}
&\text{§ Control all systematic errors:} \\
&\quad \text{☞ Finite-volume effects } L \to \infty \\
&\quad \text{☞ Chiral extrapolations to physical } u/d \text{ quark masses } m_\pi \to m_\pi^{\text{phys}} \\
&\quad \text{☞ Extrapolation to the continuum limit (lattice spacing } a \to 0) \\
&\quad \text{☞ Nonperturbative renormalization using the RI/SMOM scheme} \\
&\quad \text{☞ Contamination from excited states} \\
&\quad \text{☞ Statistical effects}
\end{align*} \]
**Precision Nucleon Couplings**

- Much effort has been devoted to controlling systematics
- A state-of-the-art calculation (PNDME)

<table>
<thead>
<tr>
<th>$a$ (fm)</th>
<th>$V$</th>
<th>$M_\pi L$</th>
<th>$M_\pi$ (MeV)</th>
<th>$t_{\text{sep}}$</th>
<th># Meas.</th>
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<tbody>
<tr>
<td>0.12</td>
<td>$24^3 \times 64$</td>
<td>4.55</td>
<td>310</td>
<td>8,10,12</td>
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<td>0.12</td>
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<tr>
<td>0.09</td>
<td>$32^3 \times 96$</td>
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<td>310</td>
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<td>$48^3 \times 96$</td>
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<tr>
<td>0.06</td>
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<tr>
<td>0.06</td>
<td>$96^3 \times 192$</td>
<td>3.80</td>
<td>135</td>
<td>16,18,20,22</td>
<td>52.5k</td>
</tr>
</tbody>
</table>

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Huey-Wen Lin — The Proton Mass Workshop @ ECT*
Much effort has been devoted to controlling systematics.

A state-of-the art calculation (PNDME) $a = 0.12$ fm, 310-MeV pion

Move the excited-state systematic into the statistical error

$$C^{3pt}(t_f, t, t_i) = |\mathcal{A}_0|^2 \langle 0 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_0(t_f - t_i)}$$

$$+ \mathcal{A}_0 \mathcal{A}_1^* \langle 0 | \mathcal{O}_\Gamma | 1 \rangle e^{-M_0(t - t_i)} e^{-M_1(t_f - t)}$$

$$+ \mathcal{A}_0^* \mathcal{A}_1 \langle 1 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_1(t - t_i)} e^{-M_0(t_f - t)}$$

$$+ |\mathcal{A}_1|^2 \langle 1 | \mathcal{O}_\Gamma | 1 \rangle e^{-M_1(t_f - t_i)}$$

No obvious contamination between 0.96 and 1.44 fm separation.

$O_\Gamma = \gamma_\mu \gamma_5$

$O_\Gamma = 1$

$O_\Gamma = \sigma_{\mu\nu}$
§ Much effort has been devoted to controlling systematics

§ A state-of-the art calculation (PNDME) $a = 0.09$ fm, 310-MeV pion

Move the excited-state systematic into the statistical error

$$C^{3pt}(t_f, t, t_i) = |\mathcal{A}_0|^2 \langle 0 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_0(t_f-t_i)}$$

$$+ \mathcal{A}_0 \mathcal{A}_1^* \langle 0 | \mathcal{O}_\Gamma | 1 \rangle e^{-M_0(t-t_i)} e^{-M_1(t_f-t)}$$

$$+ \mathcal{A}_0^* \mathcal{A}_1 \langle 1 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_1(t-t_i)} e^{-M_0(t_f-t)}$$

$$+ |\mathcal{A}_1|^2 \langle 1 | \mathcal{O}_\Gamma | 1 \rangle e^{-M_1(t_f-t_i)}$$

Much stronger effect at finer lattice spacing!

Needs to be studied case by case
Systematic Control

§ Much effort has been devoted to controlling systematics
§ A state-of-the-art calculation (PNDME)

Statistical effect

$a = 0.06\ \text{fm}, \ 220\text{-MeV pion}$

$g_T^{\text{bare}}$

2.6k

41.6k

Plots by Boram Yoon

Huey-Wen Lin — The Proton Mass Workshop @ ECT*
§ Much effort has been devoted to controlling systematics
§ A state-of-the-art calculation (PNDME)

\[ \alpha = 0.06 \text{ fm}, \text{220-MeV pion} \]

2.6k $g_s^{\text{bare}}$

\[ g_s^{\text{bare}} = \frac{2.6k}{41.6k} \]

Plots by Boram Yoon
§ Much effort has been devoted to controlling systematics
§ A state-of-the-art calculation (PNDME)

Statistical effect (worst case) \( a = 0.06 \text{ fm}, 220\text{-MeV pion} \)

2.6k \( g_A^{\text{bare}} \)

41.6k

Plots by Boram Yoon

Huey-Wen Lin — The Proton Mass Workshop @ ECT*
Much effort has been devoted to controlling systematics.

A state-of-the-art calculation (PNDME)

Robustness of the 2-state fit

\[ g_A^{\text{bare}} \]

\[ a = 0.06 \text{ fm}, \text{220-MeV pion} \]

Plots by Boram Yoon

Huey-Wen Lin — The Proton Mass Workshop @ ECT*
Much effort has been devoted to controlling systematics.

A state-of-the-art calculation (PNDME)

Extrapolate to the physical limit

\[ g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 m_\pi^2 e^{-m_\pi L} \]

First extrapolation to the physical limit of a nucleon matrix element!
Much effort has been devoted to controlling systematics.

A state-of-the-art calculation (PNDME) aims to extrapolate to the physical limit of a nucleon matrix element.

\[ g_s(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 m_\pi^2 e^{-m_\pi L} \]

First extrapolation to the physical limit of a nucleon matrix element!
Much effort has been devoted to controlling systematics.

A state-of-the-art calculation (PNDME) extrapolates to the physical limit:

$$g_S(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 m_\pi^2 e^{-m_\pi L}$$

Using conserved vector current relation:

$$(M_n - M_p)^{QCD} = (g_S/g_V)(m_u - m_d)^{QCD}_{\text{FLAG}} = 2.59(49) \text{ MeV}$$
**Precision Nucleon Couplings**

**FLAG rating system**

New: excited-state rating

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Ref.</th>
<th>Publication Status</th>
<th>(N_f)</th>
<th>Chiral Extrapolation</th>
<th>Continuum Extrapolation</th>
<th>Finite Volume</th>
<th>Excited State</th>
<th>Renormalization</th>
<th>(g_T)</th>
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<tr>
<td>PNDME’15</td>
<td>This work</td>
<td>P</td>
<td>2+1+1</td>
<td>★</td>
<td>★</td>
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<td>★</td>
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<td>[30]</td>
<td>C</td>
<td>2+1+1</td>
<td>■</td>
<td>○</td>
<td>○</td>
<td>■</td>
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<td>1.11(3)b</td>
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<td>★</td>
<td>○</td>
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<td>1.037(20)c</td>
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<td>○</td>
<td>■</td>
<td>★</td>
<td>★</td>
<td></td>
<td>1.10(7)d</td>
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<td>P</td>
<td>2</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>★</td>
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<td>1.005(17)(29)e</td>
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<td>C</td>
<td>2</td>
<td>■</td>
<td>■</td>
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<td>■</td>
<td></td>
<td>1.114(46) f</td>
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<td>RBC’08</td>
<td>[32]</td>
<td>P</td>
<td>2</td>
<td>■</td>
<td>■</td>
<td>★</td>
<td>■</td>
<td></td>
<td>0.93(6) g</td>
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</table>

\(g_T\) and \(g_S\) graphs:

\[ (M_N - M_P)^{QCD} = 2.59(49) \text{ MeV} \]

**PNDME, 1506.06411; 1606.07049**
Given precision $g_{S,T}$ and $O_{BSM}$, predict new-physics scales.

Precision LQCD input ($m_\pi \rightarrow 140$ MeV, $a \rightarrow 0$)

$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$

Upcoming precision low-energy experiments

LANL/ ORNL UCN neutron decay exp’t

$|B_1 - b|_{BSM} < 10^{-3}$

$|b|_{BSM} < 10^{-3}$

CENPA: $^6$He($b_{GT}$) at $10^{-3}$

PNDME, PRD85 054512 (2012); 1306.5435; 1606.07049

$\Lambda_S > 7$ TeV

$\Lambda_T > 13$ TeV
§ Disconnected diagram

☞ Multiple ways to calculate this notorious contribution...
☞ Truncated solver, hopping-parameter expansion, hierarchical probing, ...
Strange Contribution

§ \langle N|s\bar{s}|N \rangle

Purely disconnected contribution

\[ g_S(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L} \]

\[ g_S^s = 0.48(18) \]

0.42(13) \chi_{QCD}

0.35(15) \text{ RQCD}

0.33(9) \text{ ETMC}

Yang this Monday, 1603.00827, 1703.08788
$\langle N | u\bar{u} + d\bar{d} | N \rangle$

Including the disconnected contribution

$$g_S(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$

PRELIMINARY

9.3(1.8) RQCD
~9.5(6) ETMC

1603.00827,1703.08788
Sigma Term

§ Convert to sigma term using FLAG 2+1+1f $m_{ud}$

35.5(2.2) MeV

\[ m_{ud} = \frac{1}{2}(m_u + m_d) \]

\[ m_{ud} = 3.70(17) \text{ MeV} \]

\[ m_{ud} = 3.373(80) \text{ MeV} \]

\[ m_{ud} = 3.6(2) \text{ MeV} \]
Nucleon Mass

§ Using Feynman-Hellman Theorem to get sigma term

\[ M_N(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_2' m_\pi^4 + c_3 a + c_4 m_\pi^2 e^{-m_\pi L} \]

[Graph showing data points and a fitted curve labeled PRELIMINARY]

\[ \sigma_{\pi N} = 63(8) \text{ MeV} \]
\[ M_N(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_2' m_\pi^4 + c_3 a + c_4 m_\pi^2 e^{-m_\pi L} \]

Overall trend: Direct calculation has smaller \( \sigma_{\pi N} \)

\[ \sigma_{\pi N} = 63(8) \text{ MeV} \]
Nucleon Mass

§ Sea-flavor dependence: 2f vs 2+1f

\[ M_N^{(4)}(M_{\pi}^2) = M_0 - c_1 4M_{\pi}^2 + \frac{\bar{\alpha}M_{\pi}^4}{2} + \frac{c_1 M_{\pi}^4}{8\pi^2 f_{\pi}^2} \ln \frac{M_{\pi}^2}{M_0^2} + \Sigma_{\text{loops}}^{(3+4)}(M_{\pi}^2) + O(p^5) \]

\[ \sigma_{\pi N}^{N_f=2+1} = 52 (3) (8) \text{ MeV} \]

\[ \sigma_{\pi N}^{N_f=2} = 41 (5) (4) \text{ MeV} \]

L. Alvarezz-Ruso et al, 1402.3991
Nucleon Mass

§ Sea-flavor dependence: 2f vs 2+1f vs 2+1+1f

\[ M_N^{(4)}(M_{\pi}^2) = M_0 - c_1 M_\pi^4 + \frac{\bar{\alpha} M_\pi^4}{2} + \frac{c_1 M_\pi^4}{8 \pi^2 f_\pi^2} \ln \frac{M_{\pi}^2}{M_0^2} + \Sigma_{\text{loops}}^{(3+4)}(M_{\pi}^2) + O(p^5) \]

\[ \sigma_{\pi N}^{N_f=2+1} = 52 \ (3) \ (8) \ \text{MeV} \]

\[ \sigma_{\pi N}^{N_f=2} = 41 \ (5) \ (4) \ \text{MeV} \]

L. Alvarez-Ruso et al, 1402.3991
§ On Monday, Jianwei mentioned

✧ **Quark mass contribution:**

\[
H_m = \int d^3 \vec{x} \bar{\psi} m \psi
\]

\[
M_m = \left. \frac{\langle P | H_m | P \rangle}{\langle P | P \rangle} \right|_{\text{at rest}} = b M_p
\]

✧ **Trace anomaly contribution:**

\[
H_a = \int d^3 \vec{x} \frac{9 \alpha_s}{16 \pi} \left( \mathbf{E}^2 - \mathbf{B}^2 \right)
\]

\[
M_a = \left. \frac{\langle P | H_a | P \rangle}{\langle P | P \rangle} \right|_{\text{at rest}} = (1 - b) \frac{1}{4} M_p
\]

§ \( bM_p = 2m_{ud} \langle N | u\bar{u} + d\bar{d} | N \rangle + m_s \langle N | s\bar{s} | N \rangle = 80(16) \text{ MeV} \)

\[ \Rightarrow b = 0.086(17) \]

✧ Quark-mass contribution to proton mass: 8.6%

§ Trace anomaly contribution: 22.8%
How about Nuclei?

§ Only $A \leq 4$ light nuclei are calculated on the lattice

▫ Assume isospin symmetry

$$\sigma_{Z,N} = \overline{m} \langle Z, N | \bar{u}u + \bar{d}d | Z, N \rangle = \overline{m} \frac{d}{d\overline{m}} E_{Z,N} \approx \frac{M_\pi}{2} \frac{d}{dM_\pi} E_{Z,N}$$

▫ Express in terms of the nuclear binding energies $E = AM_N - B$

▫ Impulse correction is $\delta \sigma_{Z,N} = \frac{\langle Z,N(gs)|\bar{u}u+\bar{d}d|Z,N(gs)\rangle}{A\langle N|\bar{u}u+\bar{d}d|N\rangle} - 1$

§ Lattice + physical data suggest effects are $O(10\%)$ or less

NPLQCD, 1306.6939
**Strangeness**

§ Importance of $g_S^s$

☞ Strange-quark intrinsic-spin contribution to proton

☞ Astrophysics application: the CCSN “problem”

3D explosions require $g_A^s \approx -0.2$  

Janka, Melson, & Summa (2016)

§ Global fit: $g_A^s \approx -0.1$

assumptions often used:

\[
\Delta \bar{s}(x, Q^2) = \Delta \bar{u}(x, Q^2) = \Delta \bar{d}(x, Q^2) = \frac{1}{2} \Delta s^+(x, Q^2)
\]

§ Lattice status

More players since the last Spin

Lighter pion masses
**Fundamental Questions**

Advanced computing makes it possible!

§ How does QCD bind (hyper)nuclei?

![Diagram showing binding energies of various nuclei](image)


**SU(3) flavor-symmetric quark masses**
§ Extrapolate to the physical limit

\[ g_T^d = -0.233(28), g_T^u = 0.774(66), g_T^s = 0.008(9) \]

Observation of a neutron EDM between the current limit and \( 4 \times 10^{-28} \, \text{e} \cdot \text{cm} \) would falsify the split-SUSY scenario with gaugino mass unification.
PNDME

Precision Neutron-Decay Matrix Elements

https://sites.google.com/site/pndmelmqcd/

Tanmoy Bhattacharya  Rajan Gupta  HWL  Vincenzo Cirigliano

Saul Cohen  Anosh Joseph  Yong-Chull Jang  Boram Yoon
Nucleons are more complicated than mesons because...

- **Noise issue**
  - Signal diminishes at large $t$ relative to noise

- **Excited-state contamination**
  - Nearby excited state: Roper(1440)
  - Hard to extrapolate in pion mass
  - Δ resonance nearby; multiple expansions, poor convergence...
  - Less an issue in the physical pion-mass era

- **Requires larger volume and higher statistics**
  - Ensembles are not always generated with nucleons in mind
  - High-statistics: large measurement has $65k$ samples

**The disappearance of X(750)**
Nucleons are more complicated than mesons because...

- Noise issue
  - Signal diminishes at large $t_E$ relative to noise

- Excited-state contamination
  - Nearby excited state: Roper(1440)

- Hard to extrapolate in pion mass
  - Δ resonance nearby; multiple expansions, poor convergence...
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- Requires larger volume and higher statistics
  - Ensembles are not always generated with nucleons in mind
  - High-statistics: large measurement has 65k samples

**PROCEED WITH CAUTION**
Nucleon Axial Charge

§ Summary

<table>
<thead>
<tr>
<th>$N_f = 2$</th>
<th>$N_f = 2+1$</th>
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<tbody>
<tr>
<td>1.195(33)(20)</td>
<td>PNDME '16</td>
</tr>
<tr>
<td>LHPC '12</td>
<td>LHPC '10</td>
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<td>AWR S '16</td>
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<td>COMPASS '15</td>
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§ Implications?

|$2\sigma$ might go away with greater statistics

<table>
<thead>
<tr>
<th>Lattice 2016 Prelim.</th>
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</thead>
<tbody>
<tr>
<td>RBC* 2+1f 1.15(4)</td>
</tr>
<tr>
<td>PACS* 2+1f 1.8(4)</td>
</tr>
</tbody>
</table>

§ New physics?

|$\lambda = g_A / g_V f_{NP}$ |
| $A_0 = -2(\lambda^2 - |\lambda|) / (1 + 3\lambda^2)$ |

§ Stay tuned...

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LKTIC36 2016 Prelim.
§ Neutron lifetime discrepancy?

\[ \tau_n = 980(50) \text{ s} \]

Using lattice $g_A$ and $V_{ud}$

The Situation... Today

Slide by Geoff Greene


plot courtesy K. Grammer
§ Isovector form factors

\[ \frac{G_A}{g_A} \]

Plots by Yong-Chull Jang

§ Flavor-dependent couplings, 1\textsuperscript{st} moments of PDFs, ...

\[ \frac{G_E}{g_V} \]

qEDM by Cirigliano (this afternoon)
§ Updates from Lattice 2016

Quark and gluon momentum fraction
First moment of $q/g$ parton distribution function: $\langle x \rangle_{q/g} = \int dx \times F_{q/g}(x)$.

Connected insertion: u, d.
Disconnected insertion: u, d, s, g

ETMC: [Vaquero, Thu, 17:30] $N_f = 2$ twisted mass with clover term,

$m_\pi = 131$ MeV, $L m_\pi = 3$,
$a = 0.093$ fm.
Stout smearing to reduce noise.
Approx: $2000$ (cfgs) $\times$ $100$ (sources)

Renormalisation: mixing between $\sum_q \langle x \rangle_q$ and $\langle x \rangle_g$: 1-loop to $\overline{\text{MS}}$ at 2 GeV.

$\langle x \rangle_g^{\text{bare}} = 0.318(24) \rightarrow \langle x \rangle_g^{\overline{\text{MS}}} = 0.320(24)$,

$\langle x \rangle_u + \langle x \rangle_d + \langle x \rangle_s \rangle_g^{\overline{\text{MS}}} = 0.72(11)$

Momentum sum satisfied: $\sum_q \langle x \rangle_q + \langle x \rangle_g = 1.04(11)$

Consistent with $\chi$QCD quenched calculation [Deka, 1312.4816].

Also computed: $g_A^{u,d}$, $g_T^{u,d}$, $g_S^{u,d}$. 

Sara Collins, Lattice 2016