Fluctuations and the Phase diagram of strongly interacting matter

A. Bzdak, VK, N. Strodthoff: arXiv:1607.07375
A. Bzdak, VK, V. Skokov: arXiv:1612.05128
A. Bzdak, VK: arXiv:1707.02640
J. Steinheimer, A. Bzdak, and VK: in preparation
What we know about the Phase Diagram

Lattice QCD:
$T_c \sim 155$ MeV
pseudo-critical line up to $O(\mu^2)$
pressure (EoS) up to $O(\mu^4)$

Nuclear Liquid-Gas

Theory, Measurements

$T$}

155 MeV

$\sim 920$ MeV

$\mu$
What we “hope” for

Cross over transition

155MeV

Nuclear Liquid-Gas

~920 MeV
Is there a critical point?
Nothing you cannot find in LA...
Cumulants and phase structure

What we always see....

What it really means....

“T\(_c\)” \(\sim\) 160 MeV
Derivatives

0\textsuperscript{th} order

1\textsuperscript{st} order

3\textsuperscript{rd} order

5\textsuperscript{th} order

T_c

T_c
How to measure derivatives

At $\mu = 0$:

$$Z = \text{tr} \, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z} \text{tr} \, \hat{E} \, e^{-\hat{E}/T + \mu/T \hat{N}_B} = - \frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left( - \frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left( - \frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left( - \frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS
How to measure derivatives

At $\mu = 0$:

$$Z = \text{tr} \ e^{-\hat{\mathcal{E}}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z} \text{tr} \ \hat{\mathcal{E}} \ e^{-\hat{\mathcal{E}}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left( -\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left( -\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left( -\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS
Cumulants of Baryon number measure the chem. pot. derivatives of the EOS
Cumulants of (Baryon) Number

\[ K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle \]

\[ K_1 = \langle N \rangle, \quad K_2 = \langle N - \langle N \rangle \rangle^2, \quad K_3 = \langle N - \langle N \rangle \rangle^3 \]

Cumulants scale with volume (extensive): \( K_n \sim V \)

Volume not well controlled in heavy ion collisions

Cumulant Ratios: \( \frac{K_2}{\langle N \rangle}, \quad \frac{K_3}{K_2}, \quad \frac{K_4}{K_2} \)
Simple model

Change degrees of freedom at phase transition

\[ \langle N \rangle = \text{dof}(\mu) e^{\mu/T} \int d^3 p e^{-E/T} \]
\begin{align*}
\frac{K_2}{\langle N \rangle} & \quad \text{blue} \\
\frac{K_3}{K_2} & \quad \text{green} \\
\frac{K_4}{K_2} & \quad \text{red}
\end{align*}

\begin{align*}
\langle N \rangle / e^{\mu / T}
\end{align*}
\[
\frac{K_2}{\langle N \rangle}, \quad \frac{K_3}{K_2}, \quad \frac{K_4}{K_2}
\]

\[
\mu - \mu_c
\]

\[
\langle N \rangle / e^{\mu/T}
\]

\[
\mu - \mu_c
\]
\[
\frac{K_2}{\langle N \rangle} \quad \frac{K_3}{K_2} \quad \frac{K_4}{K_2}
\]

\[
\mu - \mu_c
\]

\[
\langle N \rangle / e^{\mu/T}
\]

\[
-1 \quad 0 \quad 1
\]

\[
\mu - \mu_c
\]
Second order may not be good enough

Search for critical point: Fluctuations vs $\sqrt{s_{NN}}$ and $A$

CP $\Rightarrow$ "Fluctuation hill"

No indication for critical point so far

M. Gazdzicki, Trento, Nov. 2017
What to expect from experiment?

Below “$T_c$”  

Above “$T_c$”  

Freeze out line
Latest STAR result on net-proton cumulants

X. Luo, arXiv:1503.02558

K₄/K₂ follows expectation, K₃/K₂ no so much.....
HADES sees similar behavior at lower energies

Comparison to STAR

- HADES data from unfolding method

HADES, preliminary (J. Stroth, INT, Oct 2016)
Let’s take the preliminary STAR data at face value
Further insights: Correlations

Cumulants

\[ K_n = \frac{\partial^n}{\partial \mu^n} \frac{P}{T^4} \]

\[ K_2 = \langle N - \langle N \rangle \rangle^2 = \langle (\delta N)^2 \rangle \]
\[ \rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2), \quad \text{C}_2: \text{Correlation Function} \]

\[ K_3 = \langle (\delta N)^3 \rangle \]
\[ \rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)C_2(p_2, p_3) + \rho_1(p_2)C_2(p_1, p_3) + \rho_1(p_3)C_2(p_1, p_2) + C_3(p_1, p_2, p_3) \]
From Cumulants to Correlations (no anti-protons)

Defining integrated correlations function

\[ C_n = \int dp_1 \ldots dp_n C_n(p_1, \ldots, p_n) \]

Simple Algebra leads to relation between correlations \( C_n \) and \( K_n \)

\[
\begin{align*}
C_2 &= -K_1 + K_2, \\
C_3 &= 2K_1 - 3K_2 + K_3, \\
C_4 &= -6K_1 + 11K_2 - 6K_3 + K_4,.
\end{align*}
\]

or vice versa

\[
\begin{align*}
K_2 &= \langle N \rangle + C_2 \\
K_3 &= \langle N \rangle + 3C_2 + C_3 \\
K_4 &= \langle N \rangle + 7C_2 + 6C_3 + C_4
\end{align*}
\]
Correlations near the critical point

M. Stephanov, 0809.3450, PRL 102

Scaling of Cumulants $K_n$ with correlation length $\xi$

\[ K_2 \sim \xi^2, \quad K_3 \sim \xi^{4.5}, \quad K_4 \sim \xi^7 \]

Cumulants from Correlations

\[
K_2 = \langle N \rangle + C_2 \\
K_3 = \langle N \rangle + 3C_2 + C_3 \\
K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4
\]

Consequently:

\[
C_2 \sim \xi^2, \quad C_3 \sim \xi^{4.5}, \quad C_4 \sim \xi^7
\]

Correlations $C_n$ pick up the most divergent pieces of cumulants $K_n$!
Significant four particle correlations!

Four particle correlation dominate $K_4$ for central collisions at 7.7 GeV

$K_2 = \langle N \rangle + C_2$

$K_3 = \langle N \rangle + 3C_2 + C_3$

$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$
Hades see even stronger correlations

Particle Correlations

\[
C_2 = -\langle N \rangle + K_2, \\
C_3 = 2\langle N \rangle - 3K_2 + K_3, \\
C_4 = -6\langle N \rangle + 11K_2 - 6K_3 + K_4.
\]

J. Stroth, INT, October 2016
Correlations

Based on prelim. STAR data

\[ C_n \]

\[ N_{\text{part}} \]

Due to presence of the first order phase transition. However, conclusions only can be made after carrying out careful theoretical and model studies for the dynamical evolution of the system including the physics of equation-of-state.

Experimentally, the protons and anti-protons numbers are measured with transverse momentum \(0< p_T < 70\text{ GeV/c}\) and at mid-rapidity \(|y|<0.5\).

\[ \kappa \sigma^2 \]

\[ \sqrt{s_{\text{NN}}} \text{ (GeV)} \]

\[ \text{NEGATIVE } C_2 \]

\[ 19.6 \text{ GeV} \]

\[ 0-5\% \]

\[ 5-10\% \]

\[ 70-80\% \]

\[ \text{UrQMD, 0-5\%} \]

\[ \text{STAR Preliminary} \]

\[ \text{Dip at 19.6 GeV from NEGATIVE } C_2 ! \]
Reduced correlation function

Reduced correlation function

\[ c_k = \frac{\int \rho_1(y_1) \cdots \rho_1(y_k) c_k(y_1, \ldots, y_k) \, dy_1 \cdots dy_k}{\int \rho_1(y_1) \cdots \rho_1(y_k) \, dy_1 \cdots dy_k} \quad C_k = \langle N \rangle^k c_k \]

Independent sources such as resonances, cluster, p+p:

\[ \begin{align*}
    c_k & \sim \frac{\langle N_s \rangle}{\langle N \rangle^k} \sim \frac{1}{\langle N \rangle^{k-1}} \\
\end{align*} \]

For example two particle correlations:

\[ c_2 \sim \frac{\text{Number of sources}}{\text{Number of all pairs}} = \frac{\text{Number of correlated pairs}}{\text{Number of all pairs}} = \frac{1}{\langle N \rangle} \]
Centrality dependence

Based on prelim. STAR data

$-c_2$

$10^{-1}$

$10^{-2}$

$10^{-3}$

$N_{\text{part}}$

$\sim 1/N^{0.85}$

$7.7$

$11.5$

$19.6$
Centrality dependence

Based on prelim. STAR data

$C_3$

$C_4$

$\frac{c_3}{N^2}$

$a = -1.5 \cdot 10^{-4}$

$b = +0.024$

$\frac{c_3}{N^2}$

$a = -7 \cdot 10^{-5}$

$b = +0.019$

$\frac{c_4}{N^3}$

$a = +7 \cdot 10^{-5}$

$b = -0.02$

$\frac{c_4}{N^3}$

$a = +4 \cdot 10^{-5}$

$b = -0.01$
Rapidity dependence

\[ C_k(\Delta Y) = \int_{\Delta Y} dy_1 \ldots dy_k \rho_1(y_1) \ldots \rho_1(y_k) c_k(y_1, \ldots, y_k) \]

Assume: \( \rho_1(y) \approx const. \)

short range correlations:

\[ c_k(y_1, \ldots, y_k) \approx \delta(y_1 - y_2) \ldots \delta(y_{n-1} - y_k) \]

\[ C_k(\Delta Y) \sim \Delta Y \rightarrow K_k \sim \Delta Y \]

Long range correlations:

\[ c_k(y_1, \ldots, y_k) = const. \]

\[ C_k(\Delta Y) \sim (\Delta Y)^k \]
Preliminary Star data are consistent with long range correlations.
Long range correlations

\[ C_k = \langle N \rangle^k c_k \]
\[ c_k = \text{const.} \Rightarrow K_n = K_n (\langle N \rangle) \]

NB: Data are consistent with small “repulsive” component

\[ c_2(y_1, y_2) = c_2^0 + \gamma_2 (y_1 - y_2)^2 \quad \gamma_2 > 0 \]
Energy dependence

Note: anti-protons are non-negligible above 19.6 GeV
Data are protons only
Can we understand these correlations?

- Two particle correlations can be understood by simple Glauber model + Baryon number conservation

Four particle correlations are orders of magnitudes larger in the data
Can we understand these correlations?

- Three and four particle correlations require lots of “fantasy”...
- For example, if about 40% of the nucleons are come in 8-nucleon clusters one can get near the data...

![Graph showing multi-particle correlations](image)

Proton-quartets = 8 nucleon clusters
Shape of probability distribution

\[ K_3 < \langle N \rangle \]
\[ K_4 > \langle N \rangle \]

\[ K_3 = \langle N - \langle N \rangle \rangle^3 \]
\[ K_4 = \langle N - \langle N \rangle \rangle^4 - 3 \langle N - \langle N \rangle \rangle^2 \]
Simple two component model
Simple two component model

Difficult to see in the real data with efficiency $\varepsilon=0.6$

$P(N)$

$\varepsilon=0.6$

$N$
Summary

- Fluctuations sensitive to phase structure:
  - measure “derivatives” of EOS
- Measurements are difficult
- Cumulants contain information about correlations
- Preliminary STAR data:
  - Significant four particle correlations at 7.7 and 11.5 GeV
  - Dip in $K_4/K_2$ at 19.6 GeV is due to negative two-particle correlations
  - Centrality dependence (at 7.7 GeV) indicates independent sources for $N_{\text{part}} < 150$ and “collective” correlations for $N_{\text{part}} > 200$.
  - At about the same centrality three- and four particle correlations change sign!
- New dynamics?
Summary

• Preliminary STAR data continued:
  - For central 7.7 and 11.5 GeV two and three particle correlations are negative and four particle are positive.

• Other more mundane effects may contribute
  - Fluctuations of system size ($N_{\text{part}}$)
    • May explain 2-particle correlations
    • Fail to reproduce the magnitude of 3- and 4- particle correlations

• Understanding 3- and 4 particle correlations requires “desperate measures”!
Thank You
π⁺π⁺, K⁺K⁺ and pp, 0-5% centrality

Minima in <R₂> of protons around Δy=0 at all beam energies

Point at Δy=0 reflects combination of SRC and the removal of track merging effects

SRC was not subtracted

S. Jowzaee, Quark Matter 2017
Things to consider

- Fluctuations of conserved charges ?!
- Volume Fluctuations
- Net-protons different from net-baryons
  - Isospin fluctuations
- “Stopping” fluctuations
- Higher cumulants probe the tails. Statistics!
- The detector “fluctuates”!
  - Efficiency effects
- .......

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Finite efficiency

Unfolding needed if we want to know what the true cumulants are
Tricky with a real detector
Compare Data with Lattice QCD

Chemical potential is better constrained than temperature
Compare Data with Lattice QCD

Example: “Charge” susceptibility

\[ \chi_Q = \int d^3x < \rho(x) \rho(0) > = \int d^3p < \tilde{\rho}(p) \tilde{\rho}(0) > \]

Equivalence of integrated coordinate space and momentum space correlation function

Experiment almost never integrates ALL of momentum space!

Lattice (hopefully) does integrate over all coordinate space
Correlations: Lattice vs Data

\[
\langle n(y_1)( n(y_2) - \delta(y_1 - y_2) ) \rangle = \langle n(y_1) \rangle \langle n(y_2) \rangle \left( 1 + C(y_1, y_2) \right)
\]

\[
C(y_1, y_2) \sim \exp\left( \frac{-(y_1 - y_2)^2}{2\sigma^2} \right)
\]

\[
\frac{\langle (\delta N)^2 \rangle}{\langle N \rangle} = 1 + \langle N \rangle \int_{\Delta/2}^{\Delta} C(y_1, y_2) \, dy_1 \, dy_2
\]

Alice Charge Flucts

“Lattice result”

“Charge conservation”

Charge conservation

Lattice result