Compton scattering from Protons and Light Nuclei: pinning down the nucleon polarizabilities

Judith McGovern

University of Manchester

Work done in collaboration with

Harald Grießhammer, Daniel Phillips, Gerald Feldman

Review: HG, DRP, JMcG & GF, Prog. Nucl. Part. Phys. **67** 841 (2012)

New fit of α_p, β_p: JMcG, HG, DRP, Eur. Phys. J. A **49** 12 (2013)

(1) Compton Scattering and polarisabilities

(2) Quick review of EFT

(3) State of current calculations and fits and future directions

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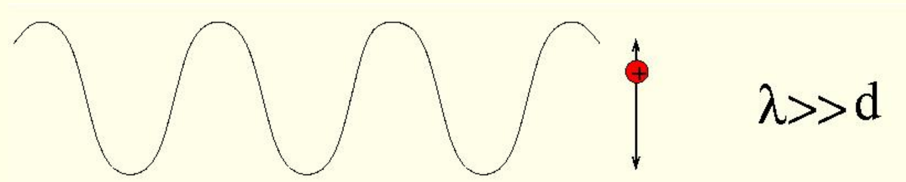
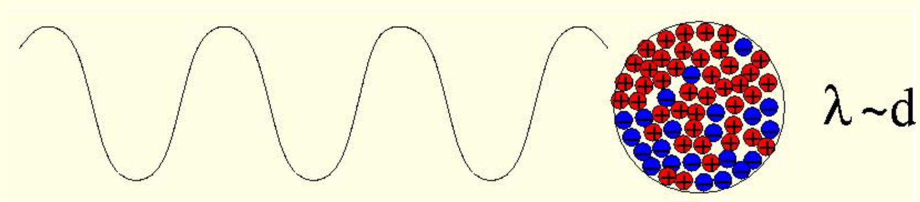
Compton Scattering

For large wavelengths, only sensitive to overall charge:



But for smaller wavelengths, the target is polarised by the electric and magnetic fields

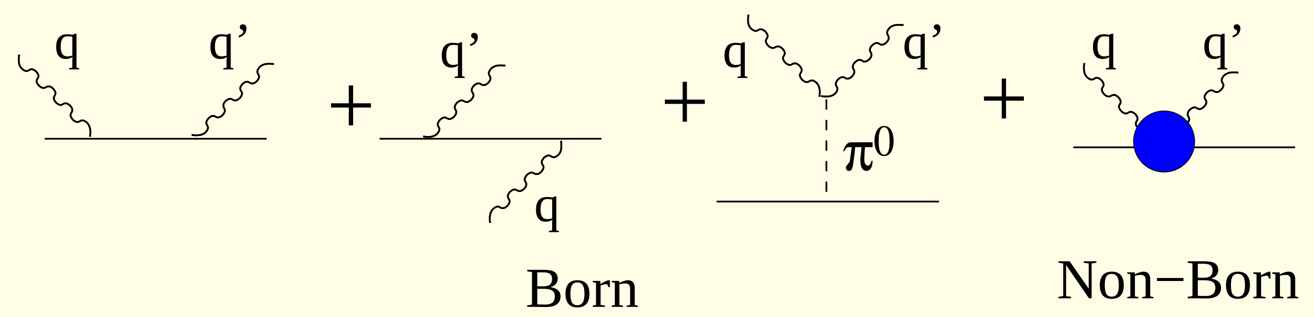


To leading order

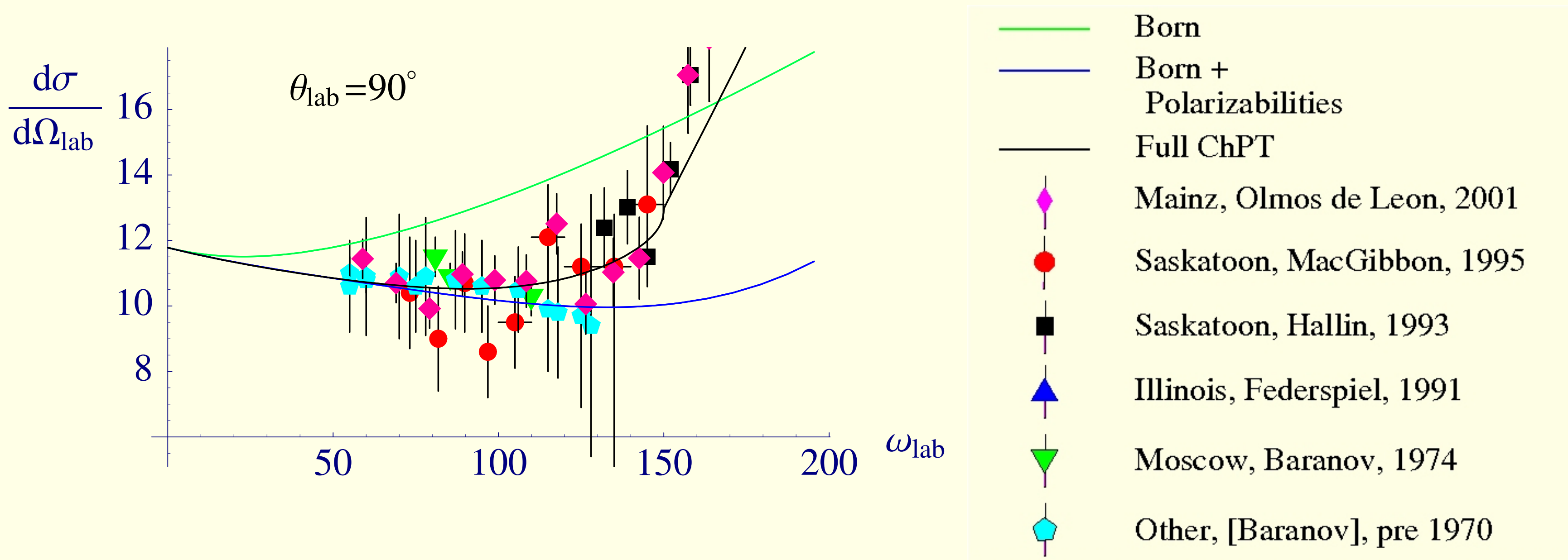
$$\begin{aligned} H_{eff} &= \frac{(\mathbf{p}-Q\mathbf{A})^2}{2m} + Q\phi - \frac{1}{2}4\pi\Big(\alpha\vec{E}^2 + \beta\vec{H}^2 \\ &+ \gamma_{E1}\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}} + \gamma_{M1}\vec{\sigma}\cdot\vec{H}\times\dot{\vec{H}} - 2\gamma_{E2}E_{ij}\sigma_i H_j + 2\gamma_{M2}H_{ij}\sigma_i E_j\Big) \end{aligned}$$

where $E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i)$ and $H_{ij} = \frac{1}{2}(\nabla_i H_j + \nabla_j H_i)$

Compton Scattering from the nucleon



The scattering amplitude has Born and non-Born pieces. The latter probe the structure of the nucleon; polarisabilities are leading signs of non-pointlike nucleons as we increase the photon energy.



Anatomy of Compton amplitude

$$
\begin{aligned}
T(\omega,z) \;=\;& A_1(\omega,z)\,(\vec{\varepsilon}\,'^{*}\cdot\vec{\varepsilon}) + A_2(\omega,z)\,(\vec{\varepsilon}\,'^{*}\cdot\hat{\vec{k}})\,(\vec{\varepsilon}\cdot\hat{\vec{k}}') \\
&+ \mathrm{i}A_3(\omega,z)\,\vec{\sigma}\cdot\left(\vec{\varepsilon}\,'^{*}\times\vec{\varepsilon}\right) + \mathrm{i}A_4(\omega,z)\,\vec{\sigma}\cdot\left(\hat{\vec{k}}'\times\hat{\vec{k}}\right)(\vec{\varepsilon}\,'^{*}\cdot\vec{\varepsilon}) \\
&+ \mathrm{i}A_5(\omega,z)\,\vec{\sigma}\cdot\left[\left(\vec{\varepsilon}\,'^{*}\times\hat{\vec{k}}\right)(\vec{\varepsilon}\cdot\hat{\vec{k}}') - \left(\vec{\varepsilon}\times\hat{\vec{k}}'\right)(\vec{\varepsilon}\,'^{*}\cdot\hat{\vec{k}})\right] \\
&+ \mathrm{i}A_6(\omega,z)\,\vec{\sigma}\cdot\left[\left(\vec{\varepsilon}\,'^{*}\times\hat{\vec{k}}'\right)(\vec{\varepsilon}\cdot\hat{\vec{k}}') - \left(\vec{\varepsilon}\times\hat{\vec{k}}\right)(\vec{\varepsilon}\,'^{*}\cdot\hat{\vec{k}})\right] .
\end{aligned}
$$

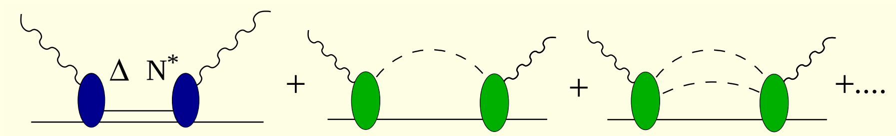
ω - photon energy, $z=\cos\theta$; Breit or cm frame

Non-Born pieces:

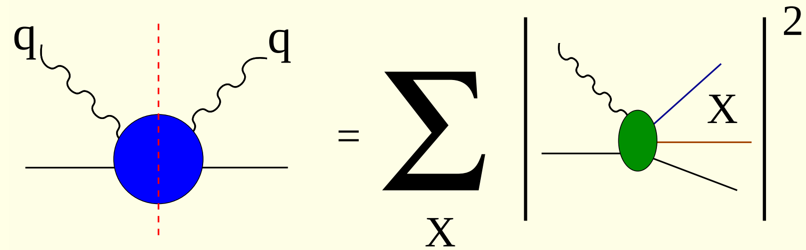
$$
\begin{aligned}
\bar{A}_1(\omega,z) &= 4\pi\,[\alpha_{E1} + z\beta_{M1}]\,\omega^2 + \ldots \\
\bar{A}_2(\omega,z) &= -4\pi\beta_{M1}\,\omega^2 + \ldots \\
\bar{A}_3(\omega,z) &= -4\pi\,[\gamma_{E1E1} + z\gamma_{M1M1} + \gamma_{E1M2} + z\gamma_{M1E2}]\,\omega^3 + \ldots \\
\bar{A}_4(\omega,z) &= 4\pi\,[-\gamma_{M1M1} + \gamma_{M1E2}]\,\omega^3 + \ldots \\
\bar{A}_5(\omega,z) &= 4\pi\gamma_{M1M1}\,\omega^3 + \ldots \\
\bar{A}_6(\omega,z) &= 4\pi\gamma_{E1M2}\,\omega^3 + \ldots
\end{aligned}
$$

If we write $\alpha_{E1}\to\alpha_{E1}(\omega)$ etc we have $l=1$ in a multipole expansion

At a hadronic level, we consider Compton scattering from the nucleon as probing its excitations and particularly its pionic cloud.



Optical theorem leads to sum rules for forward scattering



Baldin SR:

$$\alpha+\beta = \frac{1}{2\pi^2}\int_{\omega_{th}}^{\infty} \frac{\sigma_{tot}(\omega)}{\omega^2}d\omega \quad \text{and} \quad \gamma_0 = \frac{1}{4\pi^2}\int_{\omega_{th}}^{\infty} \frac{\sigma_{1/2}(\omega)-\sigma_{3/2}(\omega)}{\omega^3}d\omega$$

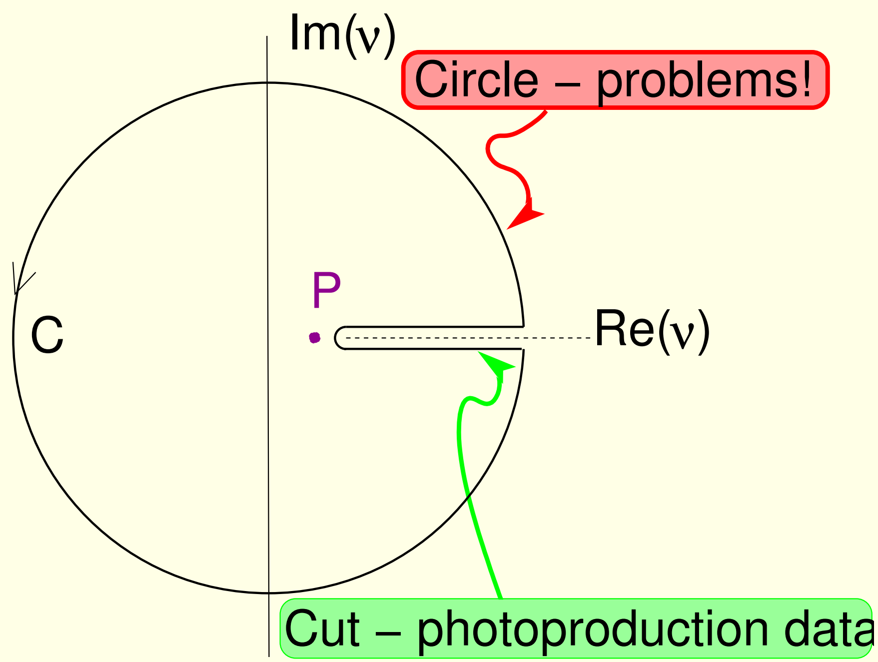
Both quite accurately evaluated for the proton:

$\alpha^{(p)}+\beta^{(p)} = (13.8\pm 0.4)\times 10^{-4}\ \mathrm{fm}^3$ Olmos de Léon *et al.* EPJA **10** 207 (2001);

$\gamma_0 = (-0.90\pm 0.08(\mathrm{stat})\pm 0.11(\mathrm{sys}))\times 10^{-4}\ \mathrm{fm}^4$ as byproduct of GDH expt. at MAMI and ELSA. Pasquini *et al.* Phys. Lett. BB **687** 160 (2010)

DR in a nutshell

Cauchy's theorem



$$\mathrm{Re}\,A_i(\nu,t) = A_i^{\mathrm{Born}}(\nu,t) + \frac{2}{\pi} P \int_{\nu_0}^{\infty} d\nu' \nu' \frac{\mathrm{Im} A_i(\nu',t)}{\nu'^2 - \nu^2},$$

PROVIDED the closure at infinity vanishes.

When it doesn't: subtract, find $A_i(0,t)$ by extrapolating from $\gamma\gamma \to \overline{p}p$

"fixed-t subtracted" Drechsel *et al.* Phys. Rep. **378** 99 (2003)

or keep circle finite and model its contribution - σ meson?

"fixed-t unsubtracted" Schumacher Prog. Part. Nucl. Phys. 55 567 (2005)

Predictions for proton polarisabilities

Chiral prediction at LO (Q^3, Bernard) and NLO (Q^4, JMcG),
also with Delta LO (δ^3, BChPT, Lensky) and NLO (δ^4, HBChPT, JMcG)

	$\alpha+\beta$	$\alpha-\beta$	γ_0	γ_π
Q^3	13.8	11.3	4.6	$[-46.4]+4.6$
Q^4	-	-	-4.0	$[-46.4]+6.1$
δ^3 B	15.1	7.4	-0.9	$[-46.4]+7.3$
δ^4 HB*	-	-	-6.8	$[-46.4]+9.7$
SR/DR	13.8 ± 0.4	10.7 ± 0.2	-0.9 ± 0.14	$[-46.4]+7.7\pm1.8$

DR: fixed-angle, Drechsel *et al.* Phys. Rep. **378** 99 (2003);

	γ_{E1E1}	γ_{M1M1}	γ_{E1M2}	γ_{M1E2}
Q^3	-5.7	-1.1	1.1	1.1
Q^4	-1.3	3.3	0.2	1.7
δ^3 B	-3.3	3.0	0.2	1.1
δ^4 HB*	-1.1	6.4	-0.4	1.9
DR(1)	-4.3	2.9	0.05	2.1
DR(2)	-3.4	2.7	0.35	1.9

DR: fixed-t,

(1) subtracted, Drechsel *et al.* Phys. Rep. **378** 99 (2003)

(2) unsubtracted Babusci *et al.* Phys. Rev. **C 58** 1013 (1998).

*In this work $\alpha-\beta=7.5\pm0.7$ and $\gamma_{M1M1}=2.2\pm0.5$ are *fit* to Compton data

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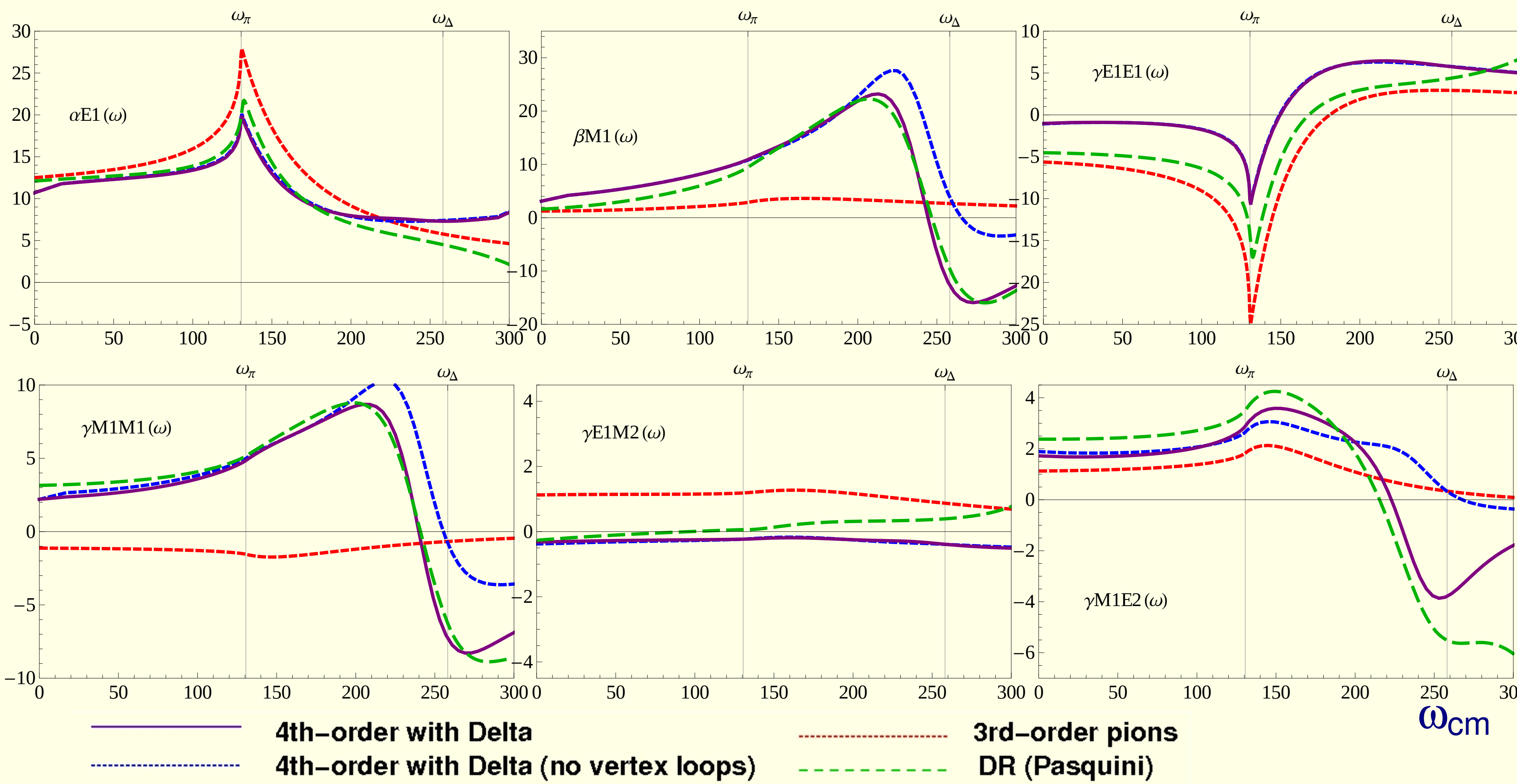
Multipoles PRELIMINARY

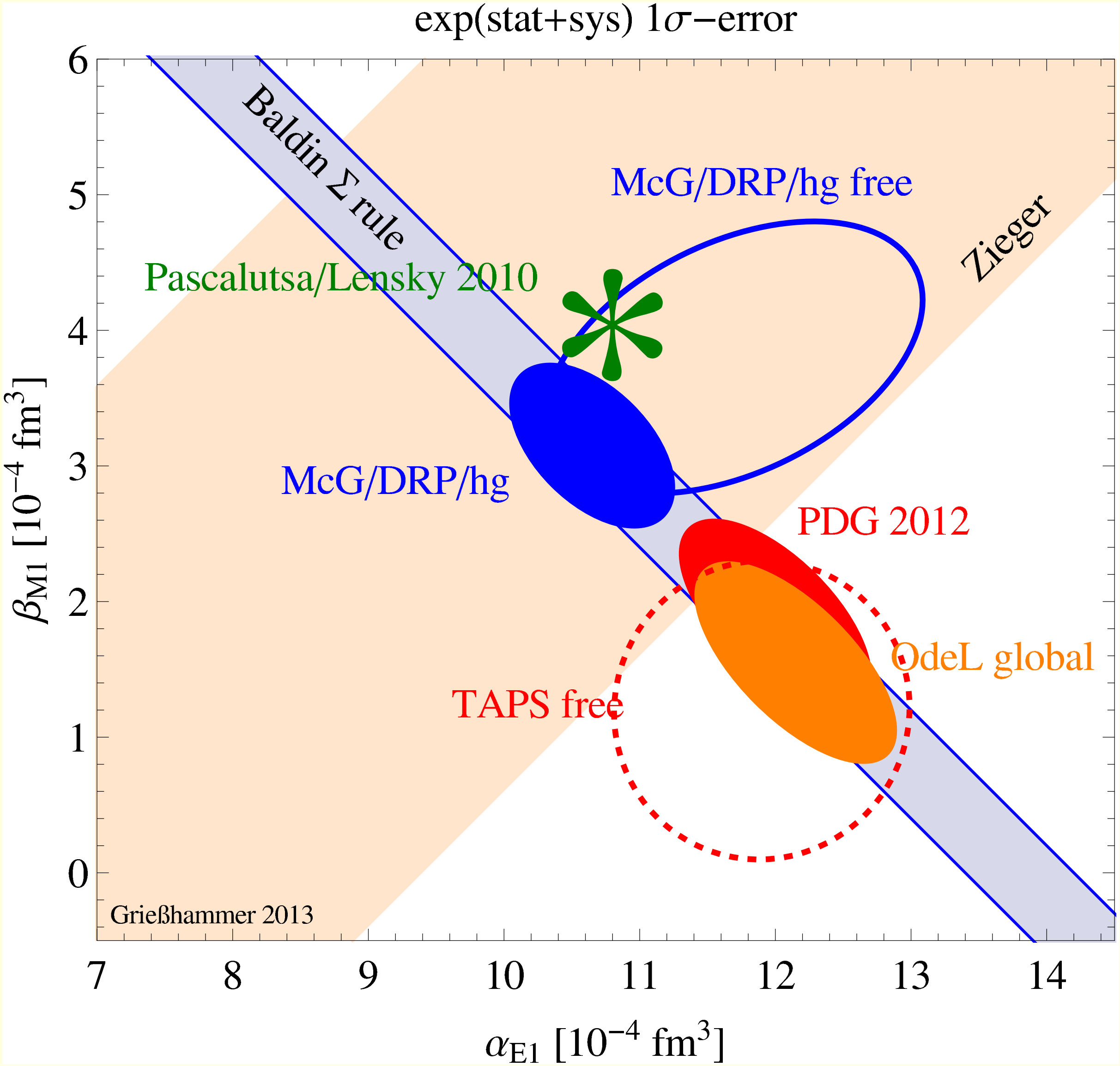
Different predictions do not fully agree on the physical origins of the polarisabilities. But chiral and DR predictions agree very well for the shape of the energy dependence of corresponding multipoles (angle-integrated amplitude)



DR: Hildebrandt *et al.* , Eur. Phys. J. A **20** 293 (2004) Chiral: JMcG *et al.* , in preparation

Our strategy: polarisabilities best obtained from Compton scattering.

Preview of results for proton



DR-based (Olmos de Léon *et al.*) and chiral (JMcG *et al.*) extractions of $\alpha - \beta$ disagree at about 2-σ (statistical) level. Why? Choice of data?

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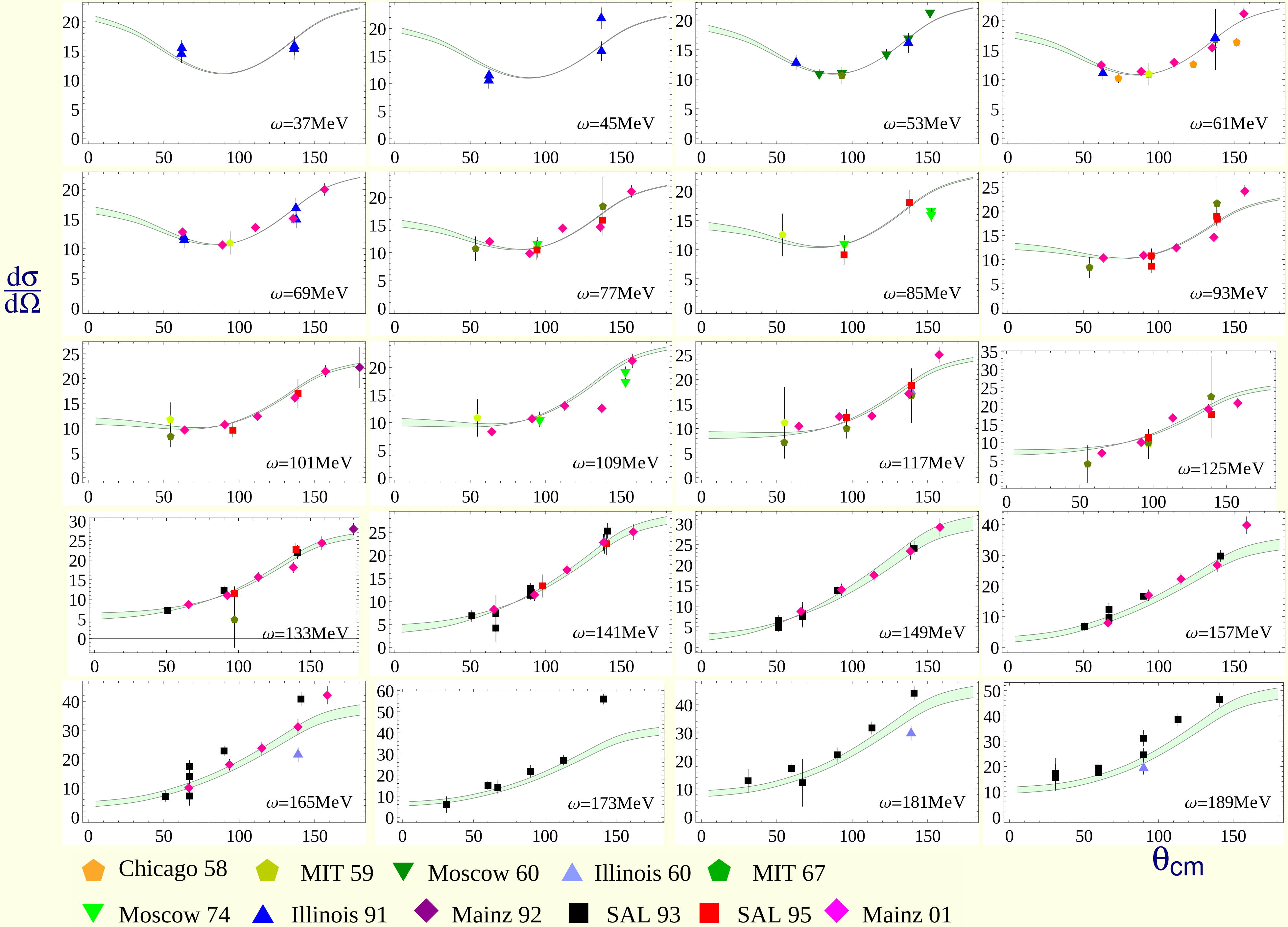
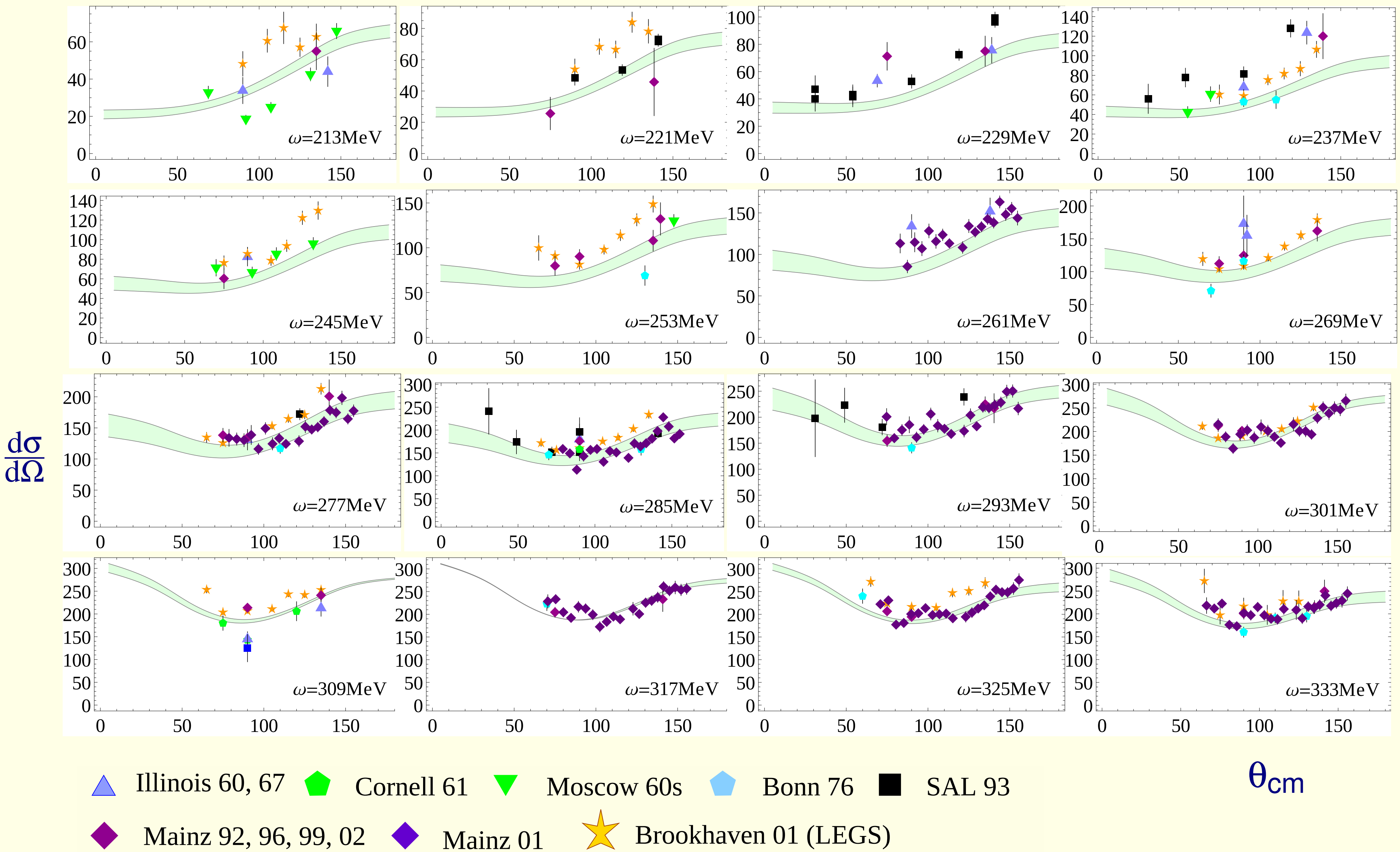
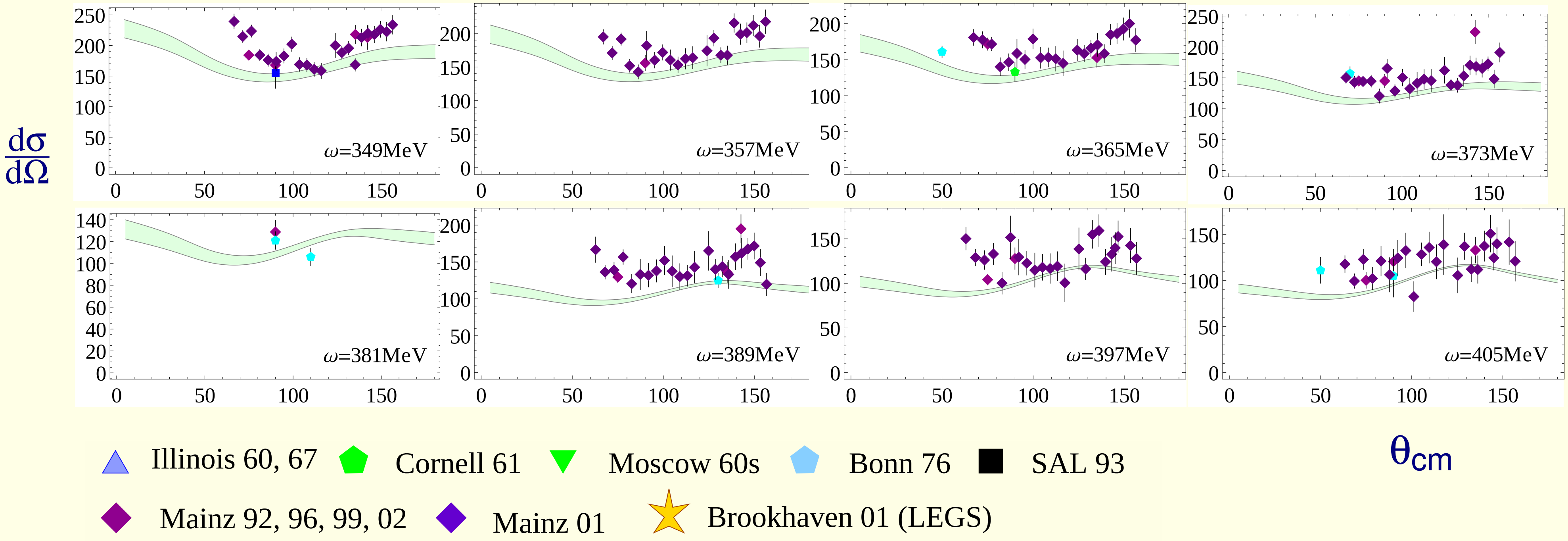
Proton data: 35-190 MeV



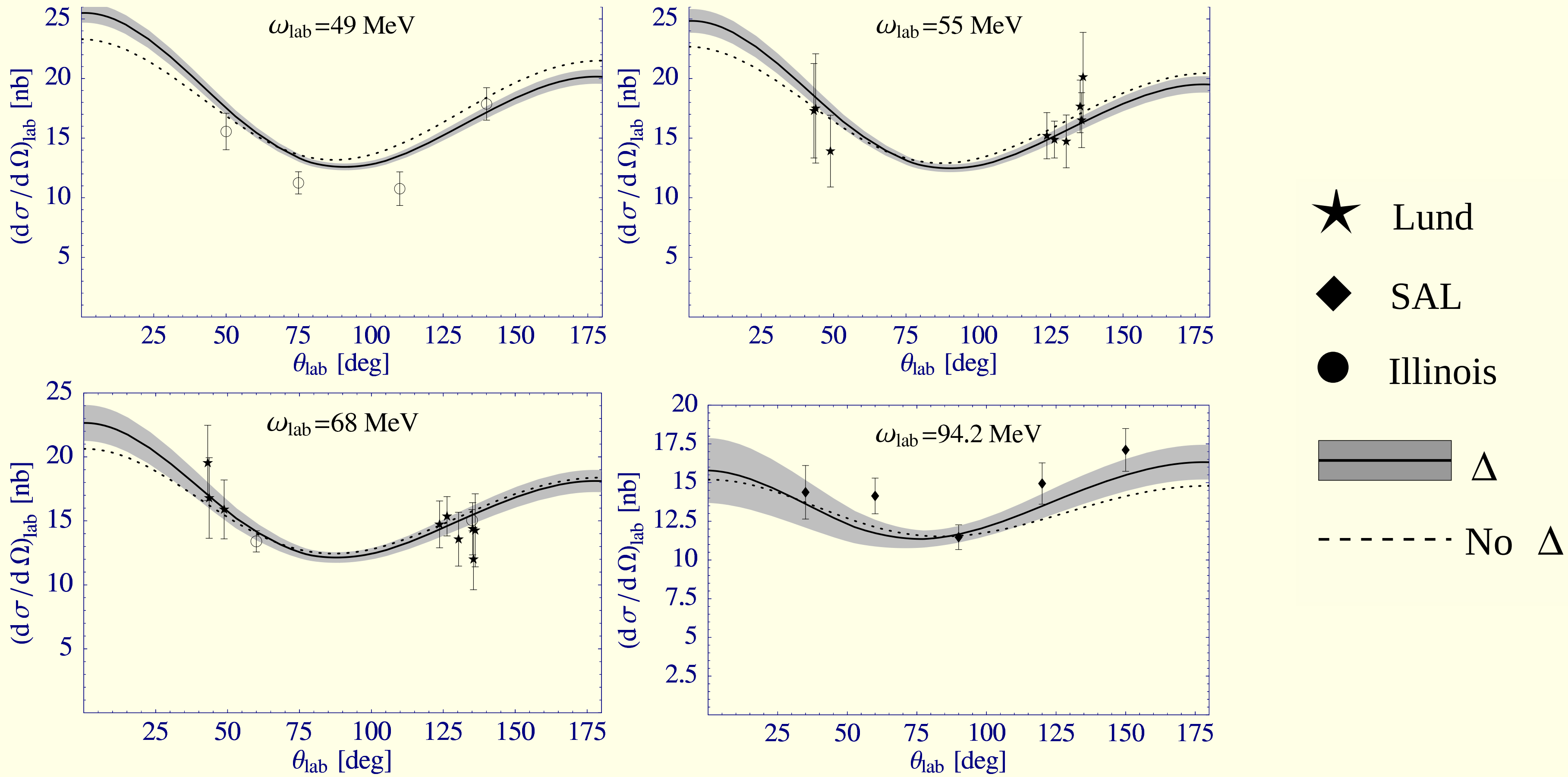
Proton data: 200-325 MeV



Proton data: 330-400 MeV



Deuteron data



source: Hildebrandt *et al.* Eur. Phys. J. A 46 (2010) 111

Much less data than for proton (difficult because of $\gamma d \to \gamma pn$).

- Illinois, $E_\gamma = 49, 69$ MeV   M. Lucas, PhD thesis, (1994)
- Saskatoon, $E_\gamma = 95$ MeV   D. L. Hornidge *et al*, PRL **84** 2334 (2000)
- Lund, $E_\gamma = 55, 66$ MeV   M. Lundin *et al*, Phys. Rev. Lett. 90 (2003) 192501

The new Lund data should expand this dramatically.

Chiral Perturbation theory

Effective field theory of QCD – relies on separation of scales

Approximate chiral symmetry of QCD (exact for massless quarks) $\Rightarrow$

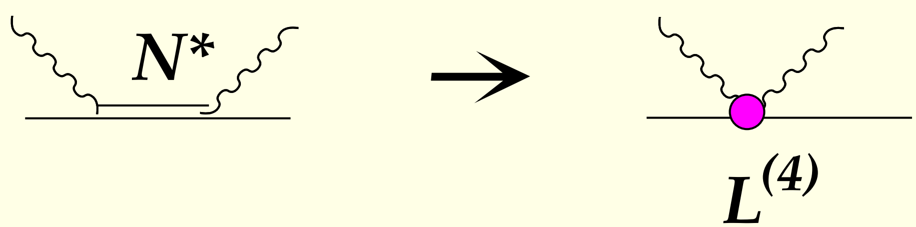
- pions are light ($m_\pi \ll m_\rho$)
- low-energy pions interact weakly with other matter ($L_{\pi NN} \propto \overline{N}\partial_\mu \pi N$).

Thus pion loops are suppressed by $\approx m_\pi^2/\Lambda^2$ where $\Lambda \approx m_\rho$.

The Lagrangian contains infinitely many terms:

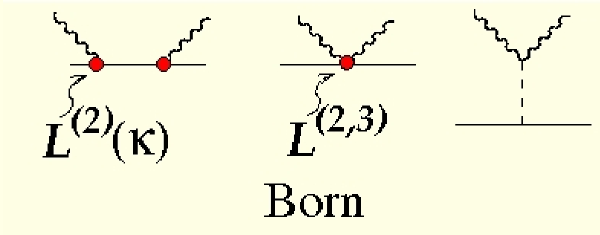
$$\mathcal{L} = \sum_n \mathcal{L}^{(n)}(c_i^{(n)})$$

Non-pionic nucleon structure shows up in low energy constants $c_i^{(n)}$, but is suppressed by power of momentum: $(k/\Lambda)^n$:



Calculations to nth order involve vertices from $\mathcal{L}^{(n)}$ and pion loops with vertices from $\mathcal{L}^{(n-2)}$; truncation errors are $\sim (k/\Lambda)^{(n+1)}$.

χPT for Compton Scattering from the nucleon

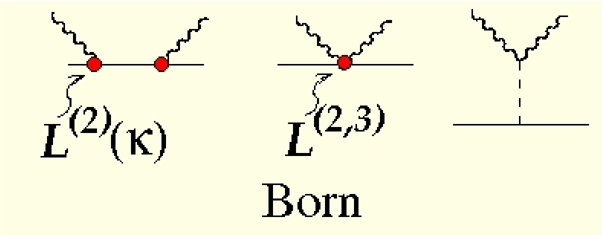


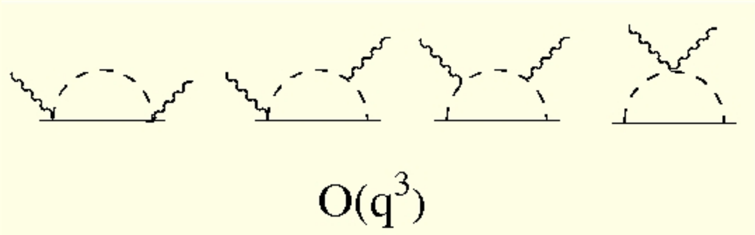
$O(Q^2)$: Thomson term (ensured by gauge and Lorentz invariance)

$O(Q^3)$: LET and pion-pole terms terms for spin-dependent for amplitudes,

,

χPT for Compton Scattering from the nucleon



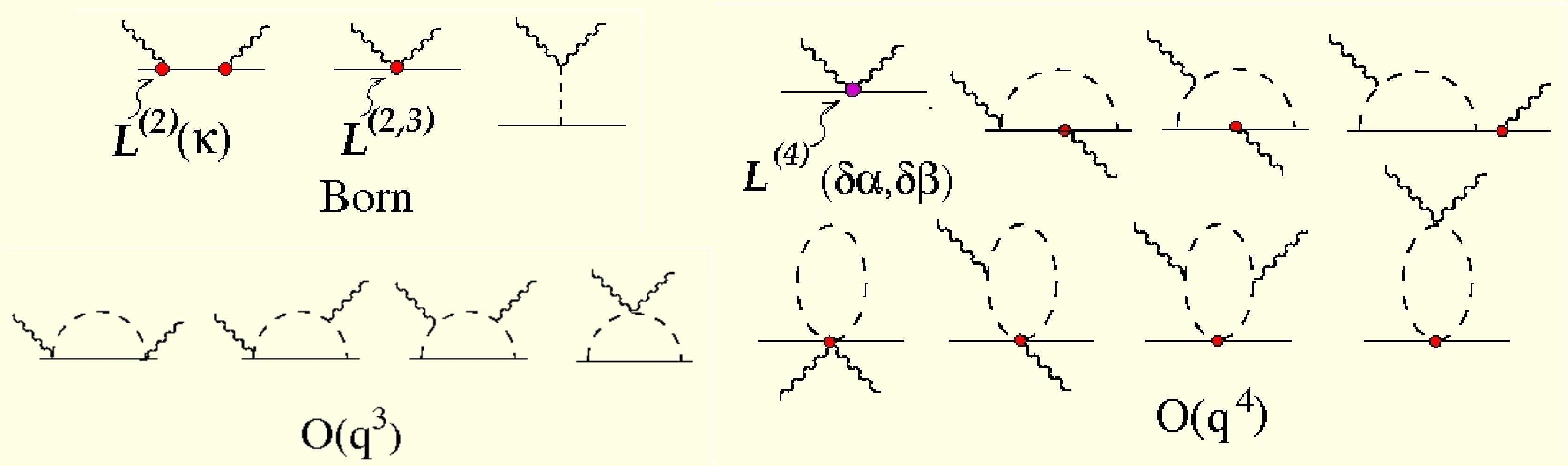


$O(Q^2)$: Thomson term (ensured by gauge and Lorentz invariance)

$O(Q^3)$: LET and pion-pole terms terms for spin-dependent for amplitudes,
+ full energy-dependent amplitude from pion loops, including predictions for polarisabilities.

,

χPT for Compton Scattering from the nucleon



$O(Q^2)$: Thomson term (ensured by gauge and Lorentz invariance)

$O(Q^3)$: LET and pion-pole terms terms for spin-dependent for amplitudes, + full energy-dependent amplitude from pion loops, including predictions for polarisabilities.

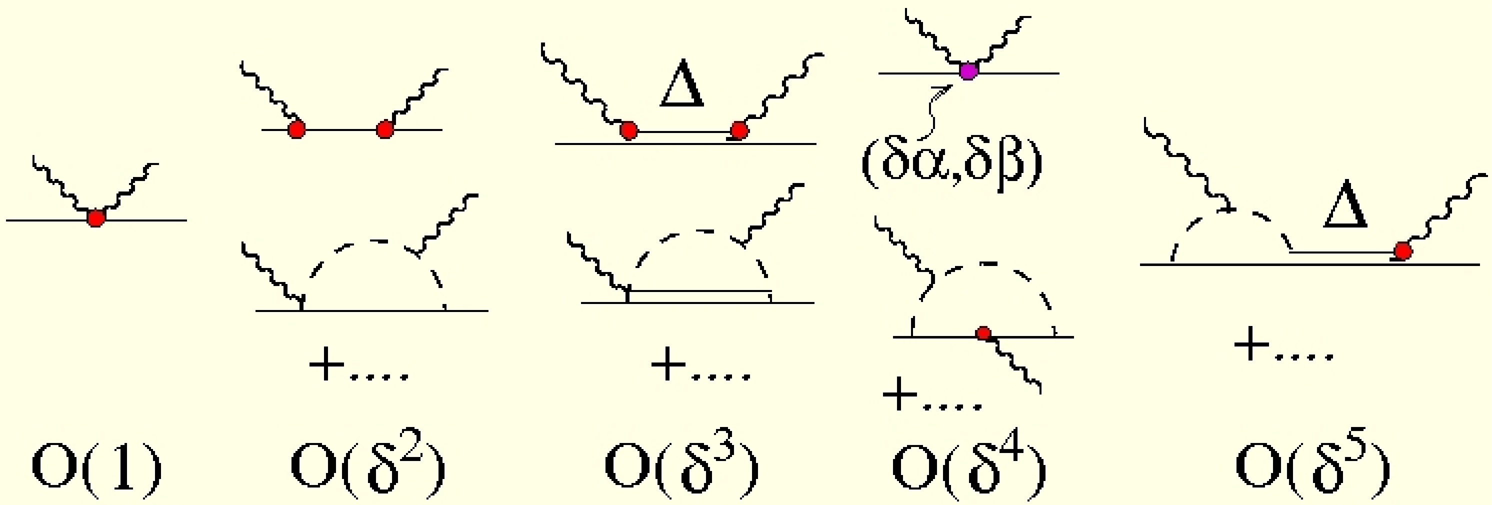
$O(Q^4)$: $1/M$ corrections and further contribution to energy-dependent amplitude BUT four undetermined LECs $\delta\alpha_p$, $\delta\beta_p$, $\delta\alpha_n$ and $\delta\beta_n$. γ_i still predicted

Including the Δ

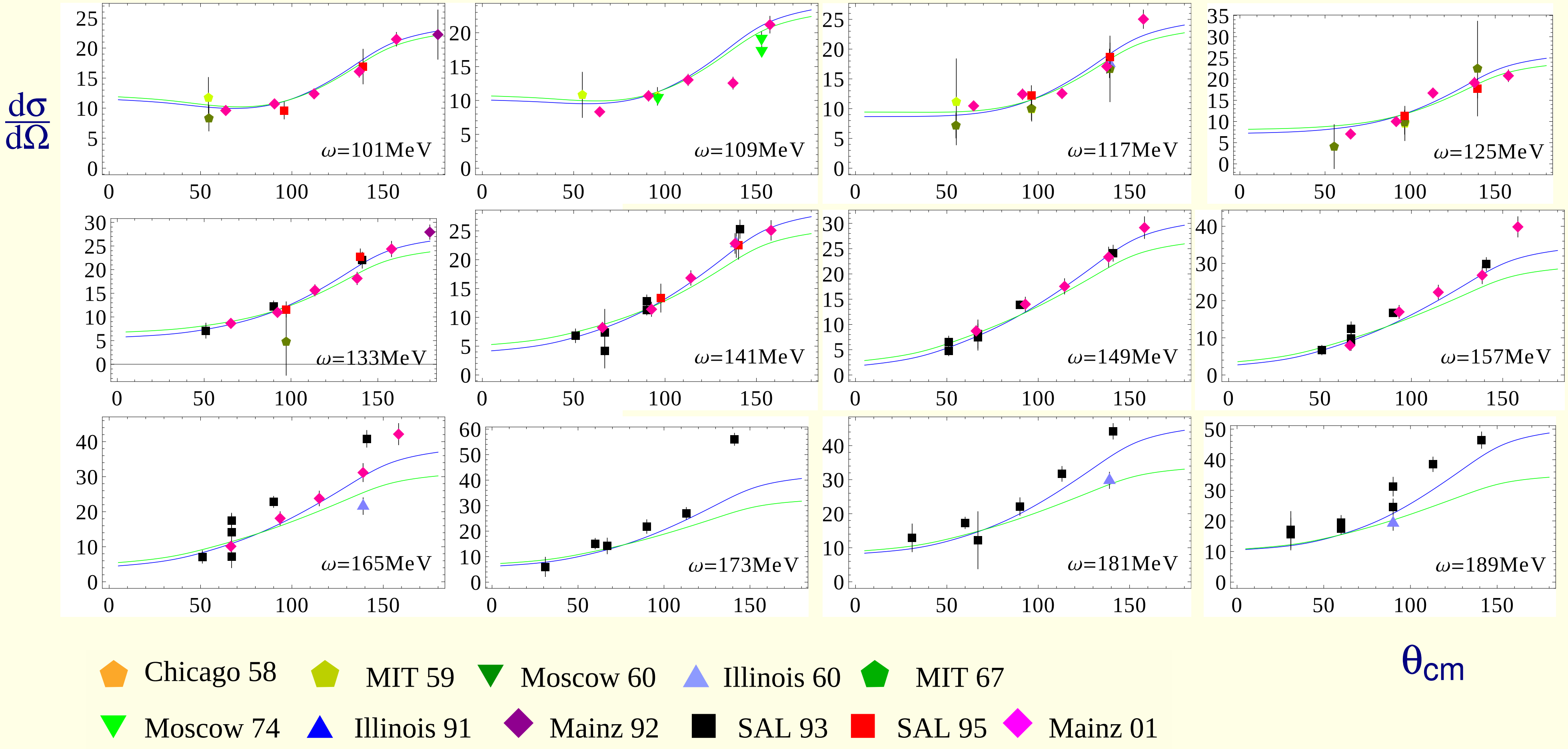
$\Delta \equiv M_\Delta - M_N \approx 271$ MeV is a rather small scale. Traditionally it is counted as $\Delta/\Lambda_\chi \sim m_\pi/\Lambda_\chi$ ("SSE"). But in Compton scattering the pion is clearly important at lower energies than the Delta.

Alternative: count $$\frac{m_\pi}{\Delta} \sim \frac{\Delta}{\Lambda_\chi} \quad \Rightarrow \quad \delta^2 \equiv \left(\frac{\Delta}{\Lambda_\chi}\right)^2 \sim \frac{m_\pi}{\Lambda_\chi}$$

Then graphs with one Δ propagator are one order of δ higher than the corresponding nucleon graphs in low energy region. Different counting in resonance region; include at least NLO in both. Pascalutsa and Phillips, Phys. Rev. C67 (2003) 055202



Effects of Delta



Δ significant from around 150 MeV upward - especially at backward angles.

$O(Q^4)$ fit w/o Δ, cut on t: $\beta \sim 3$ Beane *et al.* Phys. Lett. B **567** 200 (2003)

$O(\delta^3)$ fit with Δ, $\beta \sim 3$! Hildebrandt *et al.* Eur. Phys. J. A **20** 293 (2004)

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Fitting the proton data

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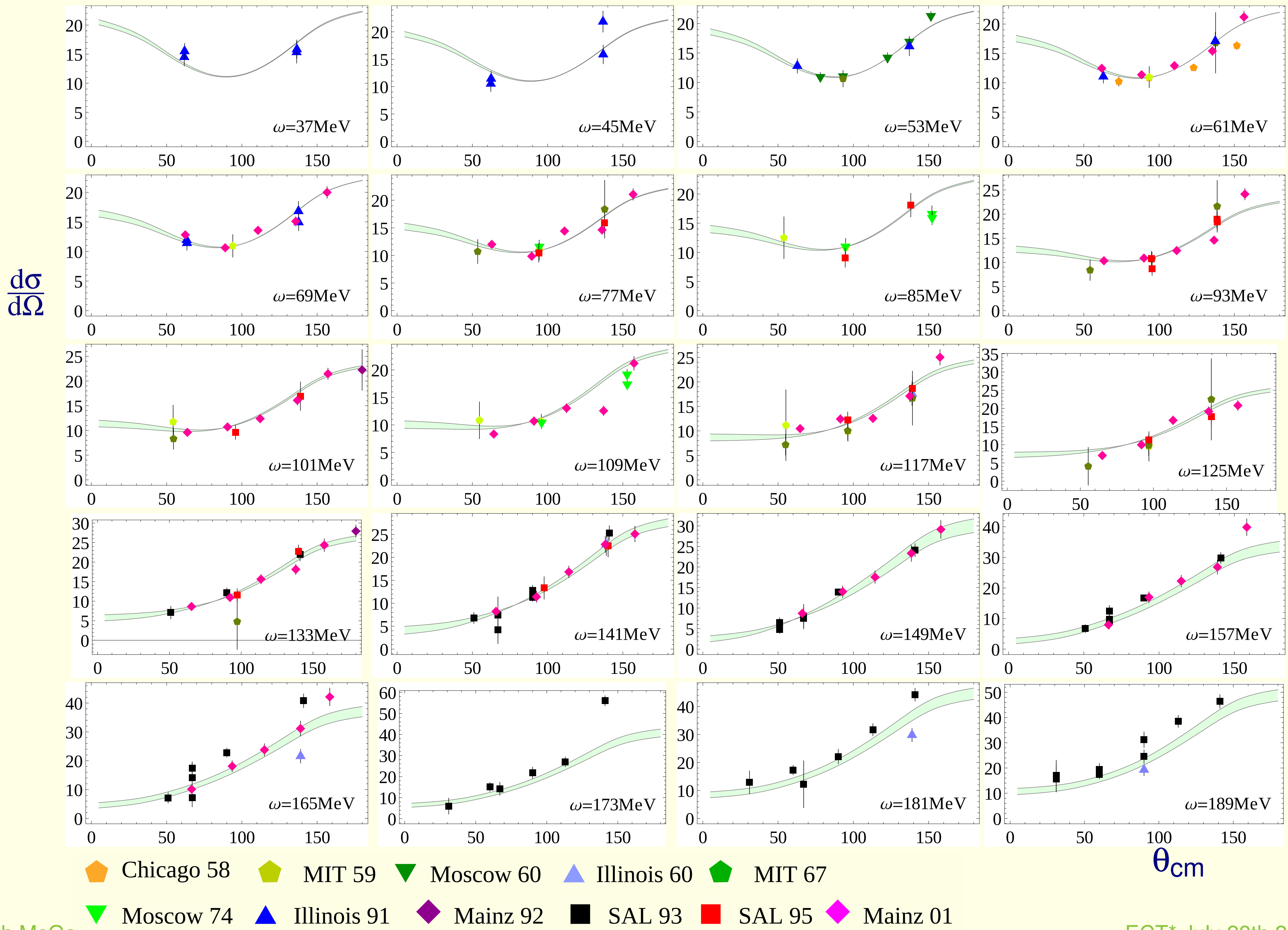
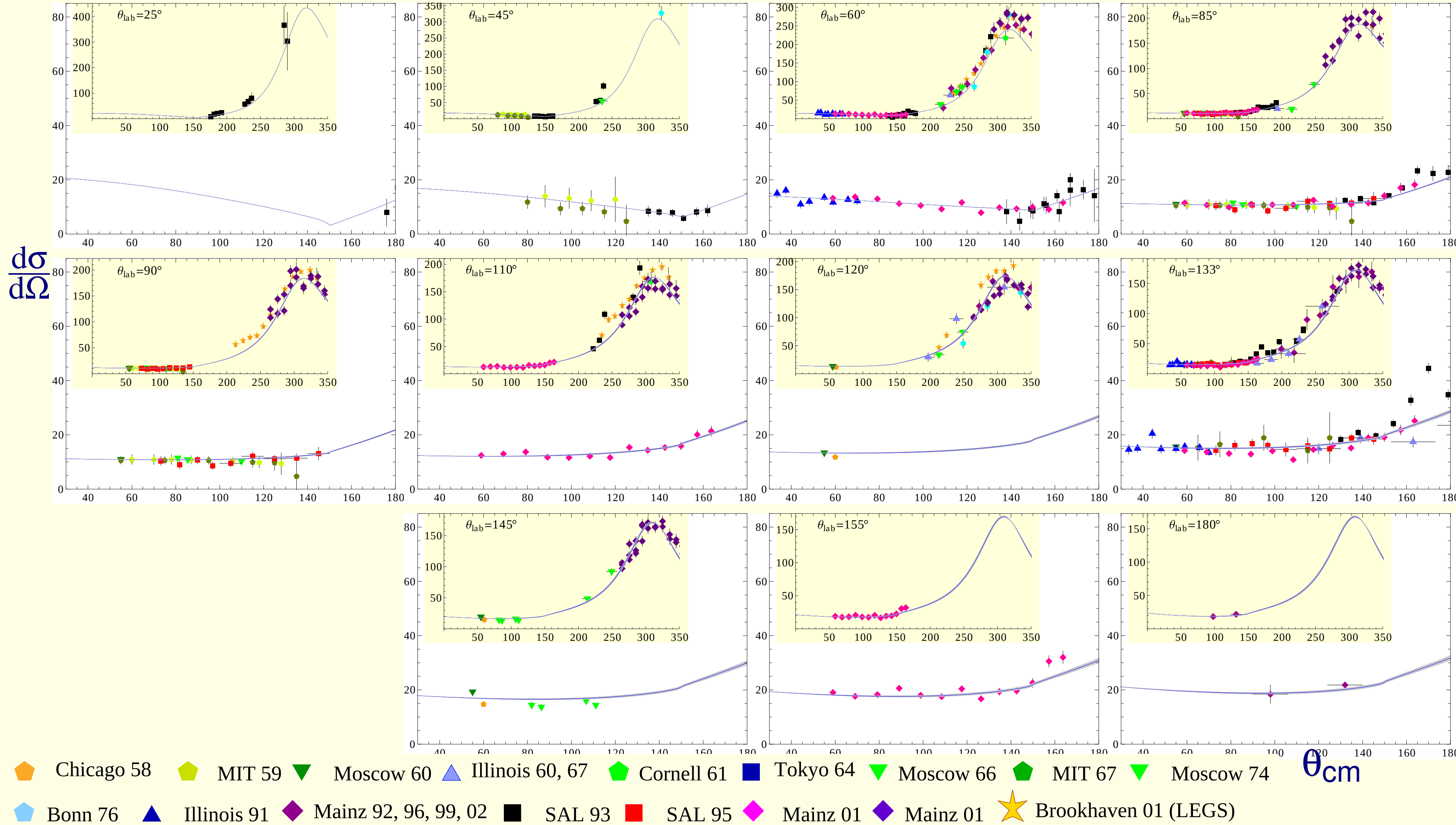




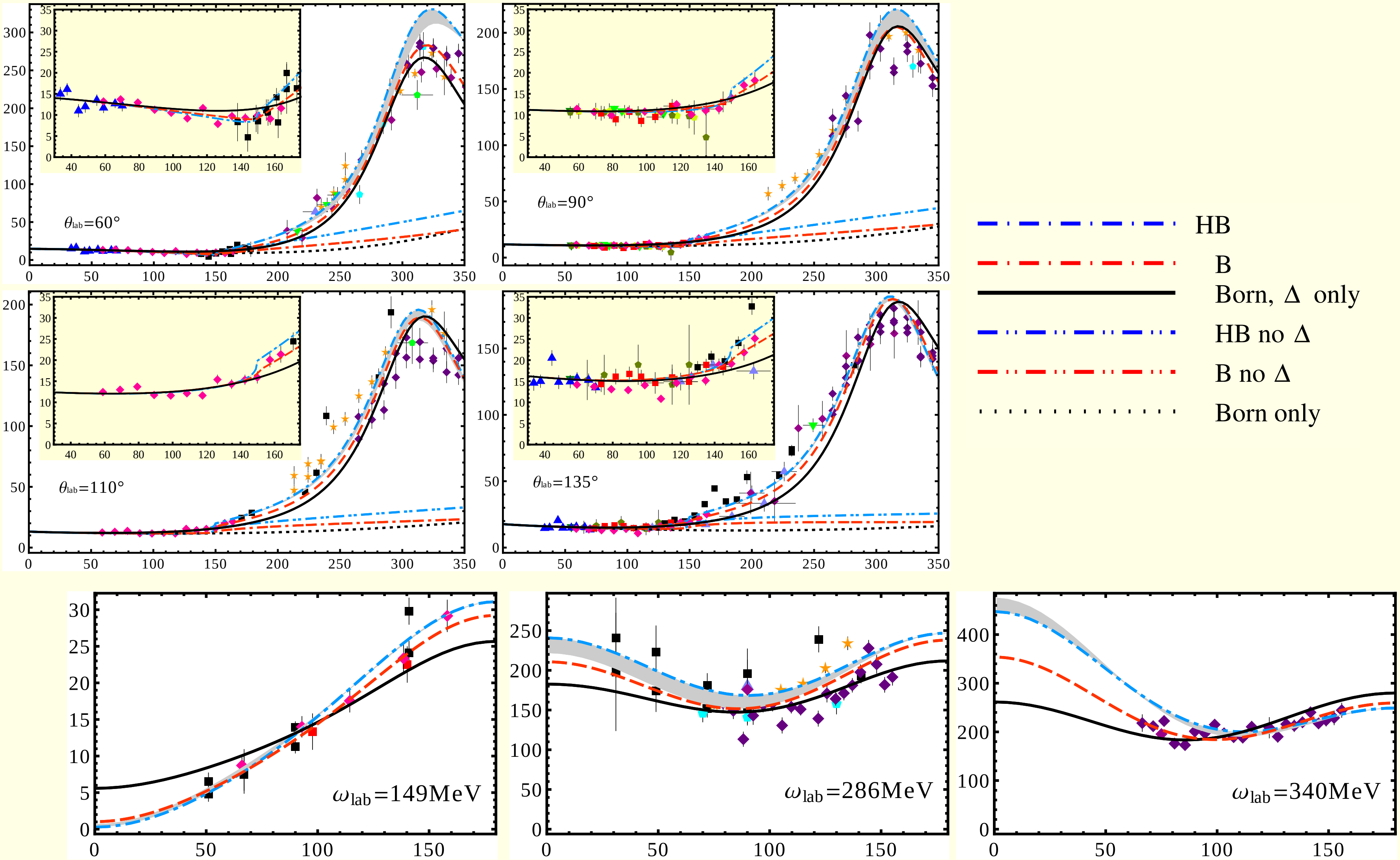
Constraining $\alpha+\beta$ with Baldin Sum rule and fitting consistent data set up to 170 MeV:

$$\alpha_p = (10.65 \pm 0.35(\text{stat}) \pm 0.2(\text{Bald}) \pm 0.3(\text{theory})) \times 10^{-4}\,\text{fm}^3$$

$$\beta_p = (3.15 \mp 0.35(\text{stat}) \pm 0.2(\text{Bald}) \mp 0.3(\text{theory})) \times 10^{-4}\,\text{fm}^3$$

also $b_1 = 3.61 \pm 0.02$, $\gamma_{M1M1} = 2.2 \pm 0.5$

Baryon vs Heavy Baryon (3rd order)



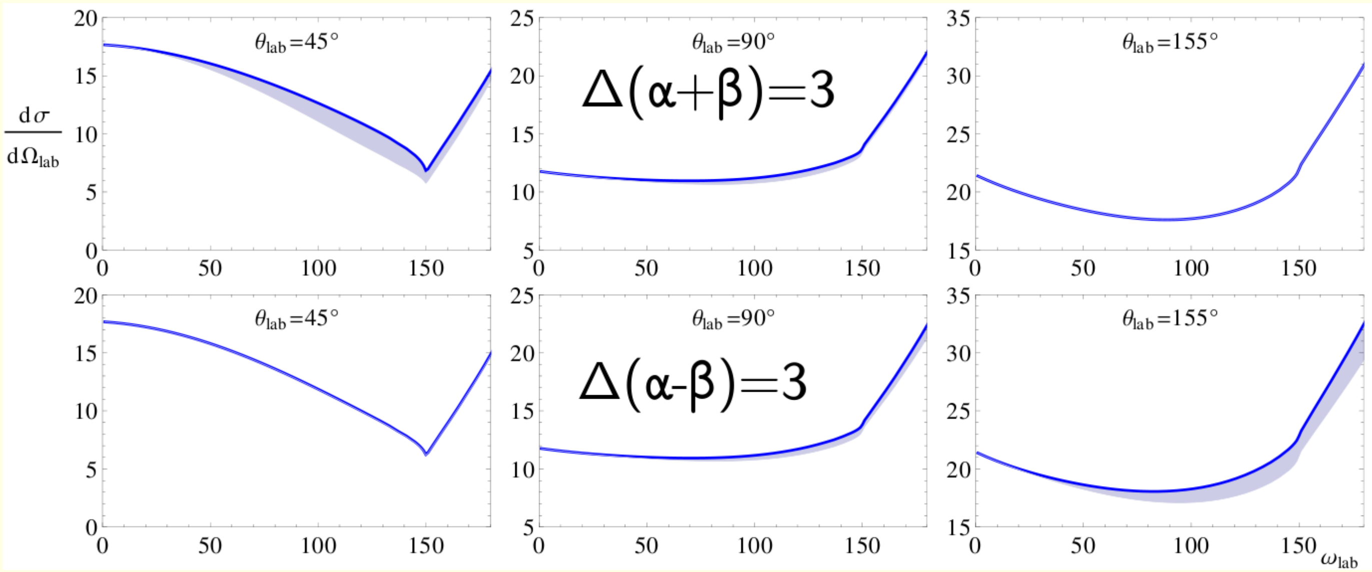
Note α and β set equal in both. Very little difference, especially below cusp.

Even better expected at 4th order (not yet done in B.)

Lensky *et al.* Phys. Rev. **C86** 048201 (2012)

We fit to low-energy data (up to 180-200 MeV), but with constraints from the higher-energy data to ensure the Δ parameters are sensible.

In spite of the amount of data, the sensitivity to the polarisabilities especially β is not very high. Magnetic response varies rapidly with energy and zero-energy value is only a small fraction of the total by 150 MeV.



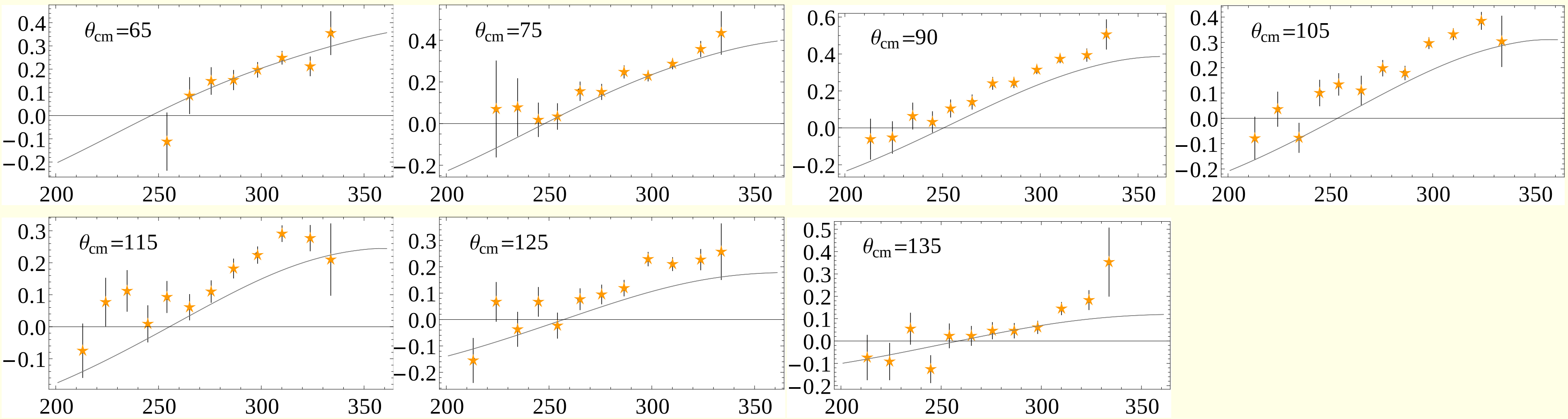
What would help

- More data in the region 160-250 MeV
- More data at forward and backward angles
- Data for polarised scattering (beam and target)

Photon beam asymmetry

LEGS data for Σ_3: Unpolarised target, photons linearly polarised parallel or perpendicular to reaction plane

$$\Sigma_3 = \frac{\sigma^{\parallel} - \sigma^{\perp}}{\sigma^{\parallel} + \sigma^{\perp}}$$

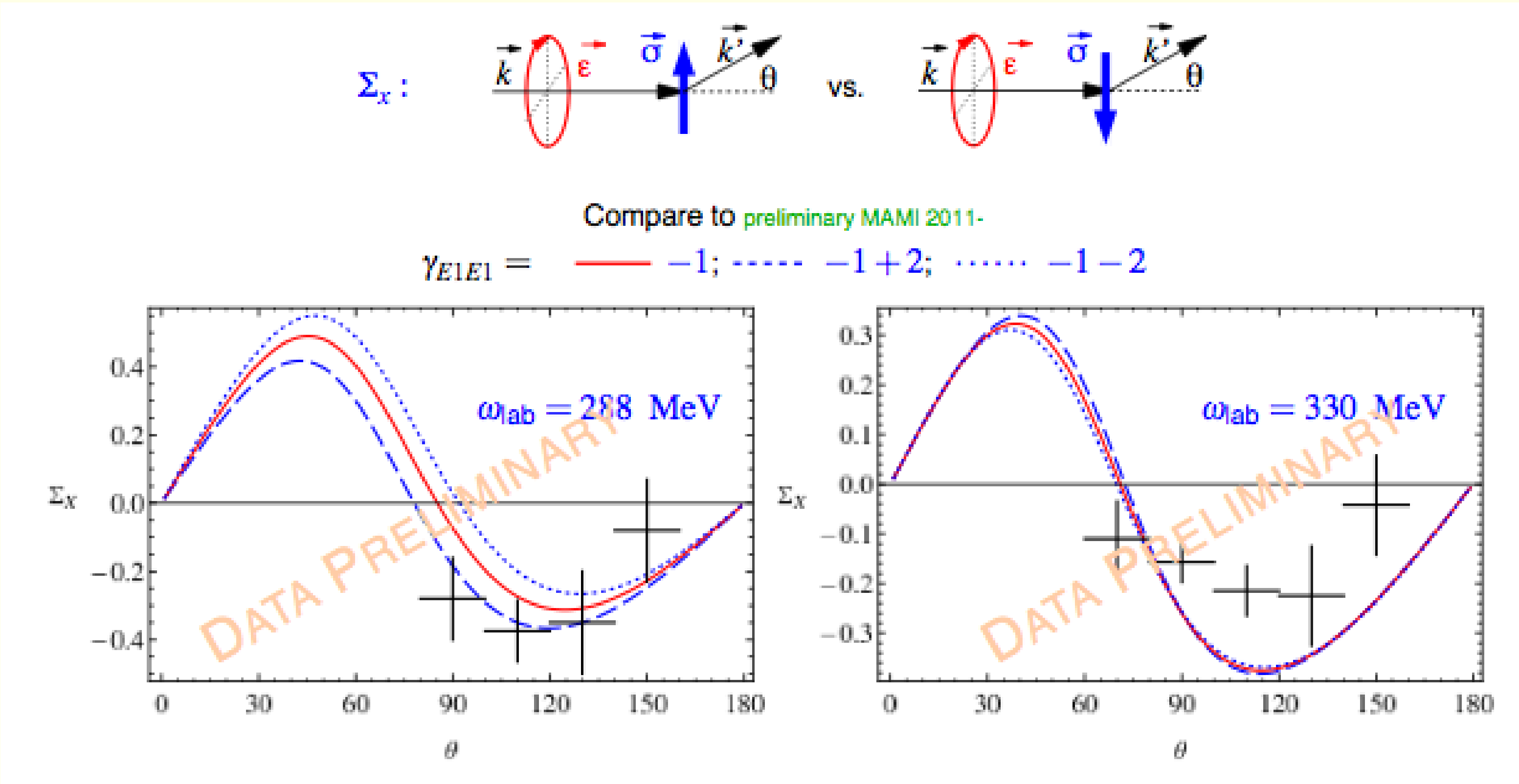


Note for the higher-energy region the calculation is only NLO...

..polarised proton

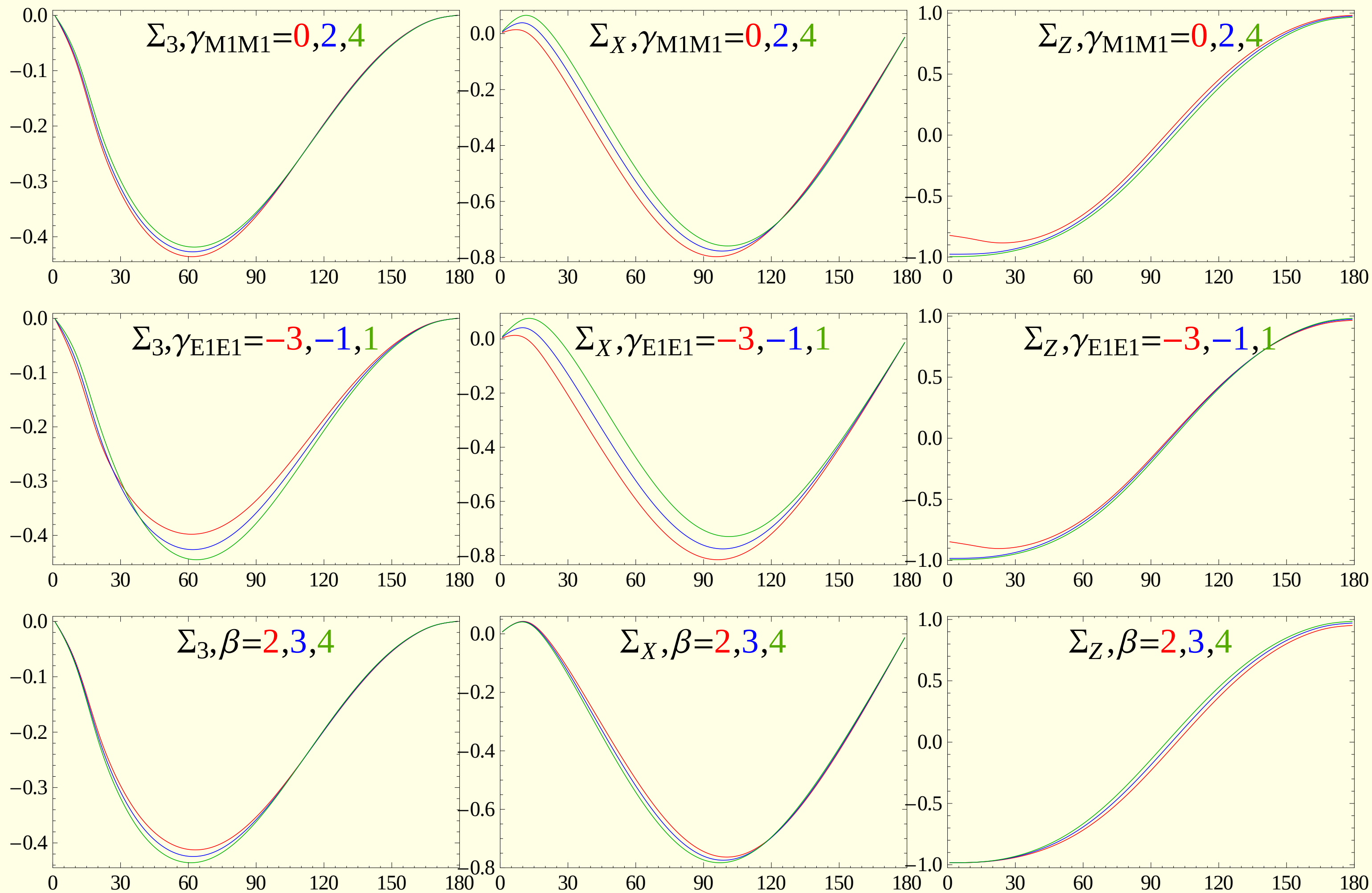
Σ_{2z} ($A^{\parallel}$) and Σ_{2x} ($A^{\perp}$): Target polarised parallel or perpendicular to reaction plane, RH or LH circularly polarised photons

$$\Sigma_{2z} = \frac{\sigma_{\parallel}^{R} - \sigma_{\parallel}^{L}}{\sigma_{\parallel}^{R} + \sigma_{\parallel}^{L}} \qquad \Sigma_{2x} = \frac{\sigma_{\perp}^{R} - \sigma_{\perp}^{L}}{\sigma_{\perp}^{R} + \sigma_{\perp}^{L}}$$



Sensitivity more reliable than overall magnitude

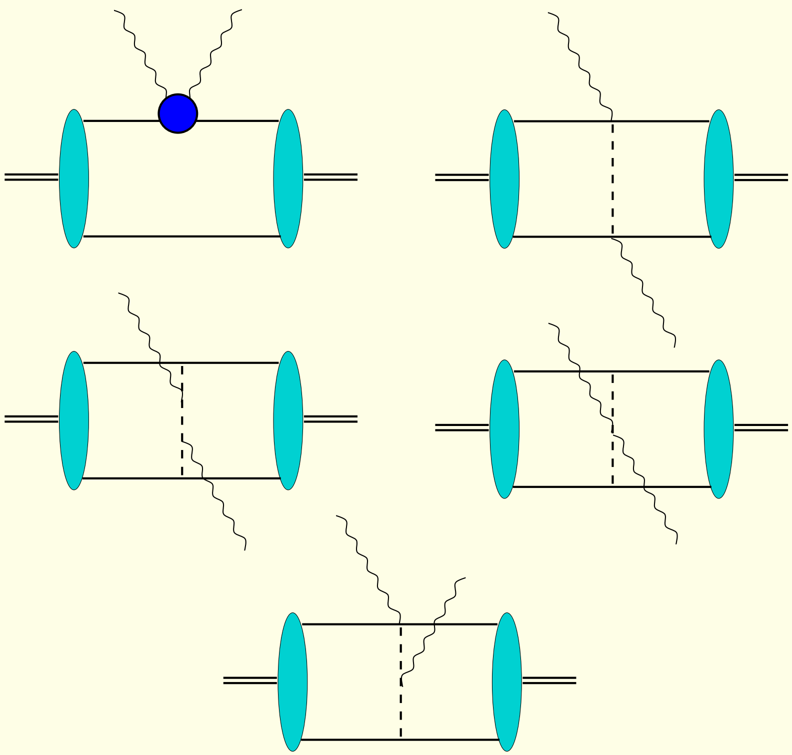
Lower energy - 150 MeV



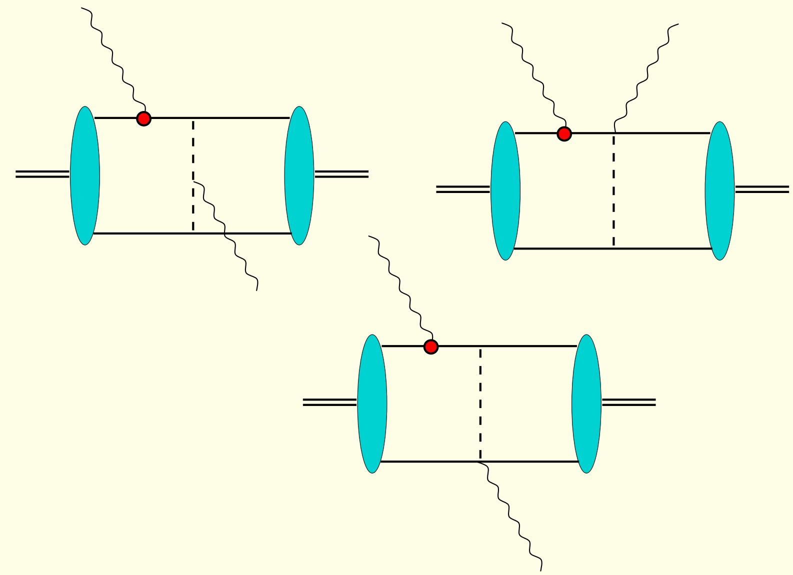
Asymmetries are large; sensitivity to γ's rises rapidly with energy.

Deuteron

Consistent treatment of one- and two-body diagrams

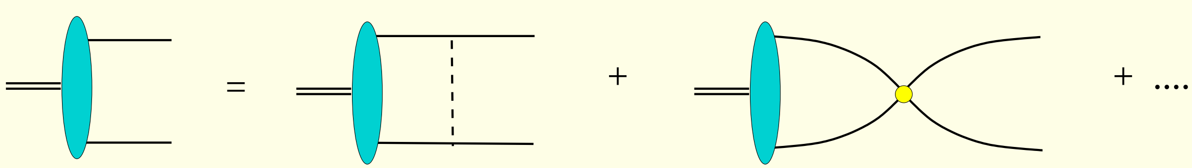


O(Q^3)



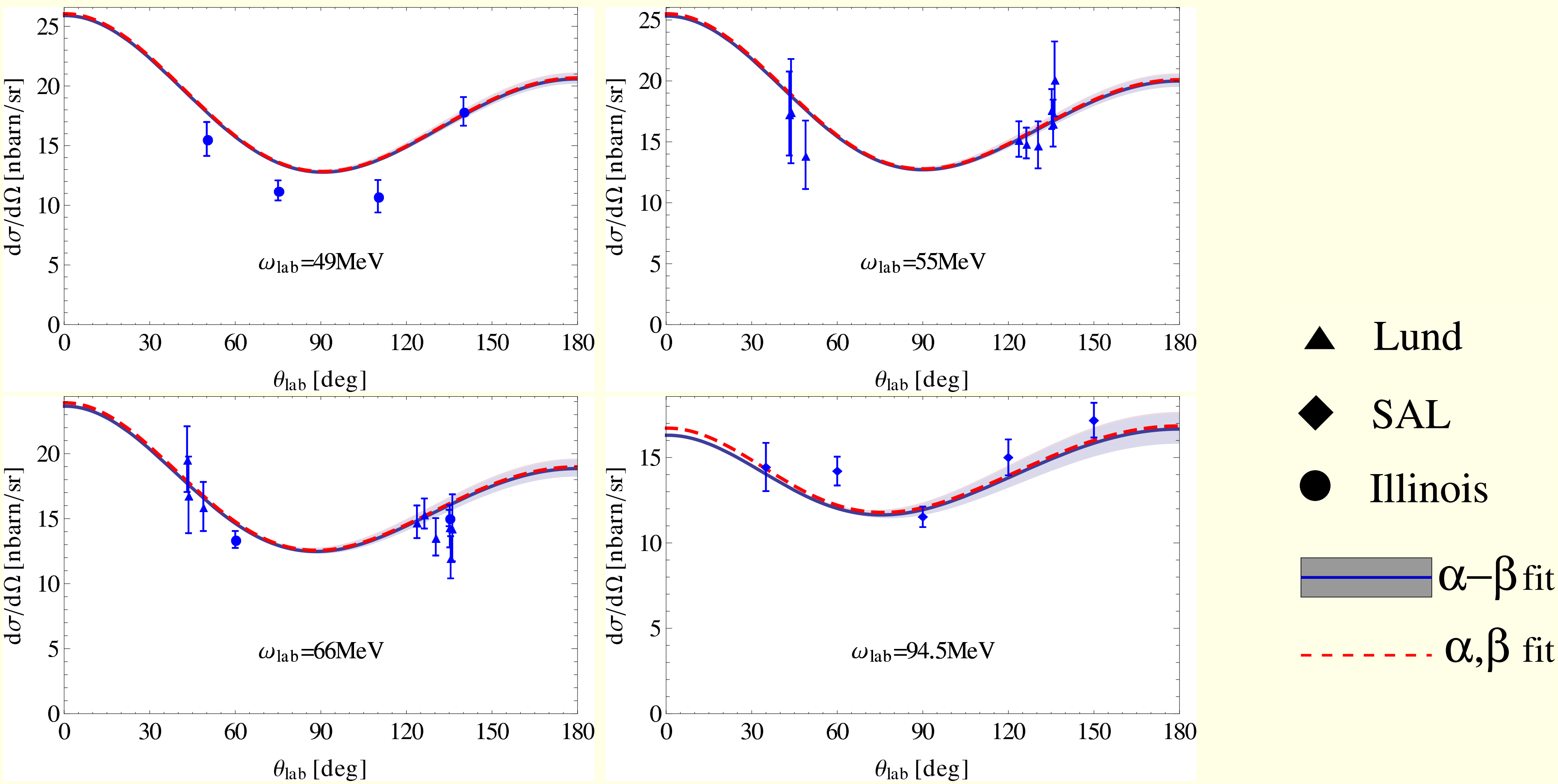
+ 1/M corrections from Lorentz invariance

O(Q^4)

where 

The Δ only enters in  at this order.

Extraction of isoscalar polarisabilities



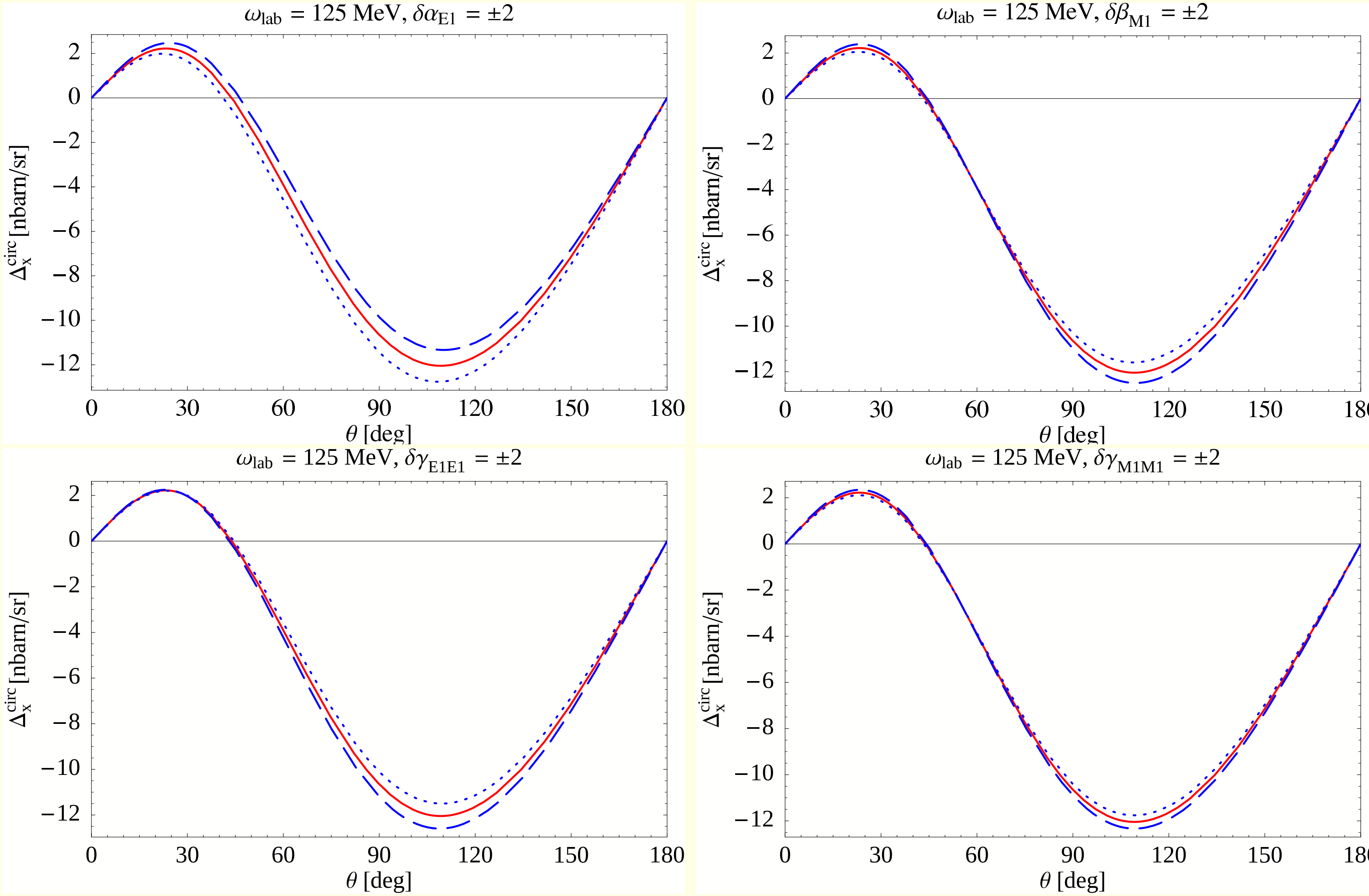
HG, JMcG, DP & GF Prog Nucl Part Phys **67** 841 (2012)

Ensuring correct Thomson limit for deuteron is important even at 50-60 MeV. So far only $O(Q^3)$; further work required to go above pion threshold.

$$\alpha_n = 11.1 \pm 1.8(\text{stat}) \pm 0.4(\text{Baldin}) \pm 0.8(\text{theory}),$$
$$\beta_n = 4.1 \mp 1.8(\text{stat}) \pm 0.4(\text{Baldin}) \pm 0.8(\text{theory})$$

Polarised scattering from deuterium

$$\Delta_x^{\text{circ}} = \frac{d\sigma}{d\Omega_{\uparrow\rightarrow}} - \frac{d\sigma}{d\Omega_{\uparrow\leftarrow}}$$

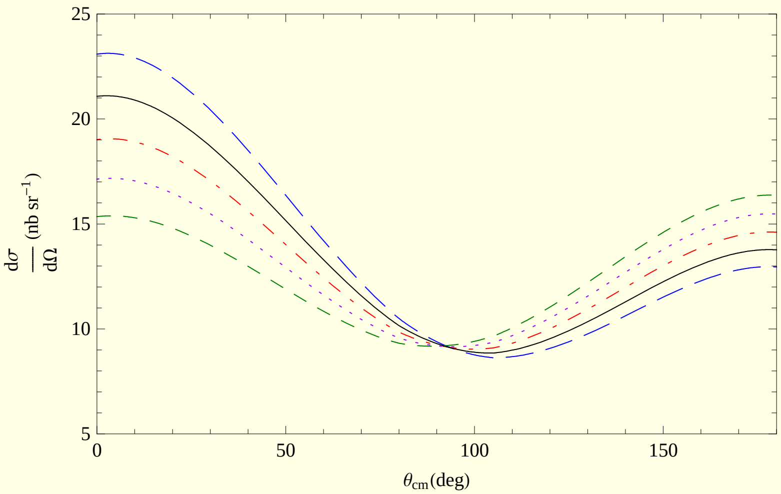


Δ included, 3rd order.

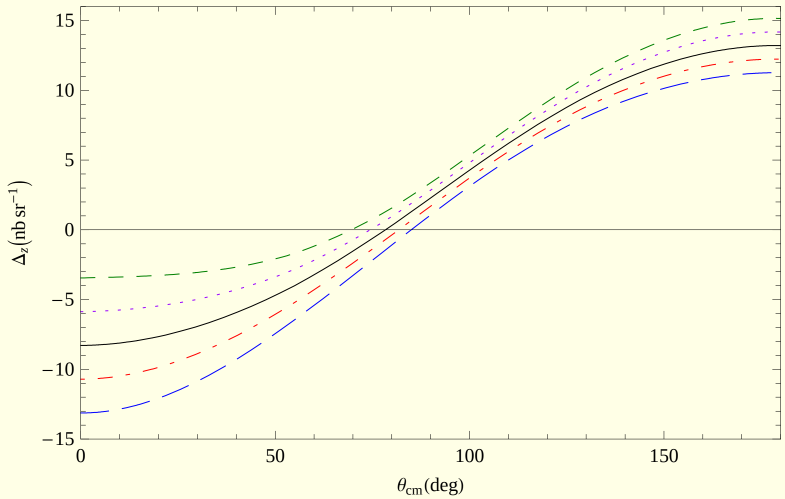
HG & Shukla, Eur. Phys. J. A **46** 249 (2010); HG, arXiv:1304.6594

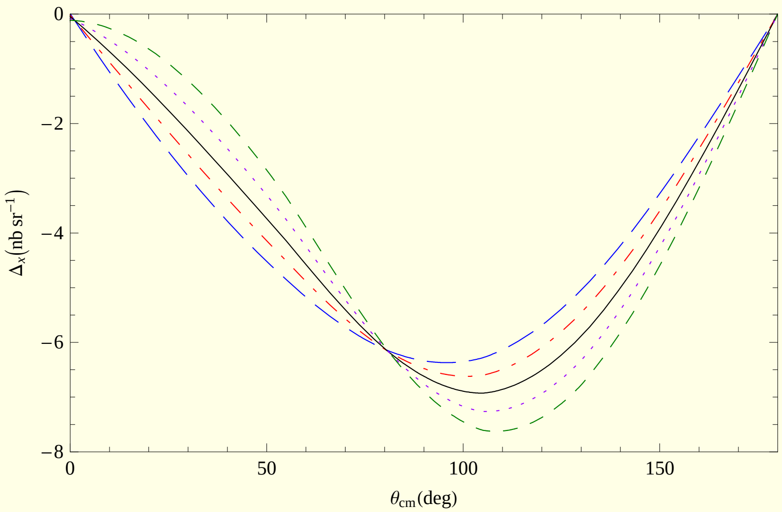
Polarised scattering from ^{3}He

Unpolarised, varying β_n

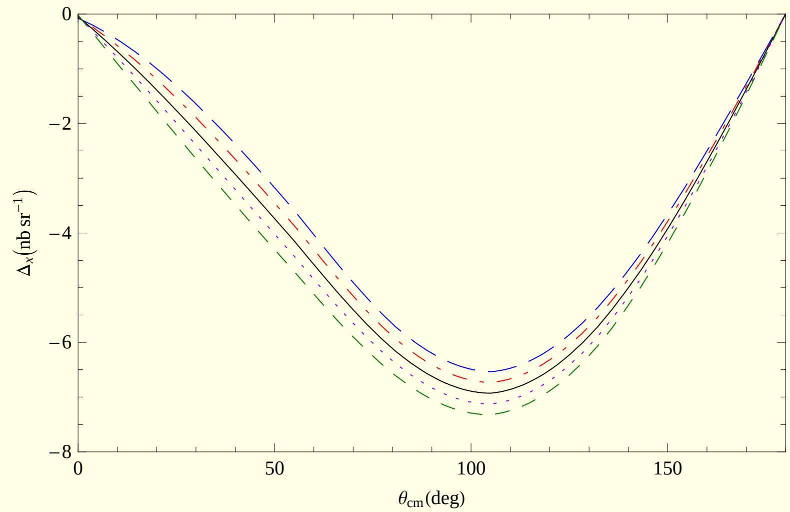


Δ_z, varying γ_{1n}





Δ_x, varying γ_{1n}



Δ_x, varying γ_{4n}

120 MeV, 3rd order, no Δ

source: Shukla, Nogga and Phillips, Nucl. Phys. A 819 (2009) 98