Light cluster production in antisymmetrized molecular dynamics

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An event of central collision of Xe + Sn at 50 MeV/nucleon (AMD calculation)

Bulk properties and dynamics
  e.g. EOS $E(\rho)$

Correlations
  e.g. clusters and fragments

Isospin dynamics, Symmetry energy

$\rho_n - \rho_p$, $n/p$, $t/3\text{He}, \ldots$
Large fraction of clusters in head-on collisions

Xe + Sn at 32 - 50 MeV/u

\[ \text{dM}_{A,Z} \]

Hudan et al., PRC67 (2003) 064613.

Partitioning of protons

- p \approx 10\%
- d, t, \(^3\)He \approx 10\%
- \(\alpha\) \approx 20\%
- \(A > 4\) \approx 60\%

(Xe + Sn at 50 MeV/u)

\[^{197}\text{Au} + ^{197}\text{Au} \text{ at 150 MeV/u}\]

Clusters (and fragments) are always the important part of the system.

The actual proton multiplicity is much smaller than the prediction by usual dynamical models.

Selected events in $^{36}$Ar + $^{58}$Ni where QP (quasi-projectile) is vaporized.

No nucleon gas has ever been produced in nuclear reactions.
Antisymmetrized Molecular Dynamics

**AMD wave function**

\[ |\Phi(Z)\rangle = \det_{ij} \exp\left\{ -\nu\left( \mathbf{r}_j - \frac{Z_i}{\sqrt{\nu}} \right)^2 \right\} \chi_{\alpha_i}(j) \]

\[ Z_i = \sqrt{\nu} D_i + \frac{i}{2\hbar \sqrt{\nu}} K_i \]

\( \nu \) : Width parameter = (2.5 fm)\(^{-2} \)

\( \chi_{\alpha_i} \) : Spin-isospin states = \( p \uparrow, p \downarrow, n \uparrow, n \downarrow \)

**Time-dependent variational principle**

\[ \delta \int_{t_1}^{t_2} \frac{\langle \Phi(Z) \mid (i\hbar \frac{d}{dt} - H) \mid \Phi(Z) \rangle}{\langle \Phi(Z) \mid \Phi(Z) \rangle} dt = 0, \quad \delta Z(t_1) = \delta Z(t_2) = 0 \]

**Equation of motion for the wave packet centroids** \( Z \)

\[ \frac{d}{dt} Z_i = \{Z_i, H\}_{PB} \text{ or } i\hbar \sum_{j=1}^{A} \sum_{\tau=x,y,z} C_{i\sigma,j\tau} \frac{dZ_{j\tau}}{dt} = \frac{\partial H}{\partial Z_{i\sigma}} \]

Motion of wave packets in the mean field

\( \mathcal{H} = \frac{\langle \Phi(Z) \mid H \mid \Phi(Z) \rangle}{\langle \Phi(Z) \mid \Phi(Z) \rangle} + \text{(c.m. correction)}, \quad H: \text{Effective interaction (e.g. Skyrme force)} \)
Antisymmetrization of Gaussian Wave Packets

An example:

When two wave packets are located with a small distance (at $-d$ and $+d$),

$$\mathcal{A}\left[e^{-\nu(r_1+d)^2} \otimes e^{-\nu(r_2-d)^2}\right] \approx \mathcal{A}\left[(1 - 2\nu d \cdot r_1)e^{-\nu r_1^2} \otimes (1 + 2\nu d \cdot r_2)e^{-\nu r_2^2}\right]$$

$$= \mathcal{A}\left[(\phi_s(r_1) - \epsilon \phi_p(r_1)) \otimes (\phi_s(r_2) + \epsilon \phi_p(r_2))\right] \quad (\epsilon = \sqrt{\nu}|d|)$$

$$= \mathcal{A}\left[\phi_s(r_1)\phi_s(r_2)\right] + \epsilon \mathcal{A}\left[\phi_s(r_1)\phi_p(r_2)\right]$$

$$- \epsilon \mathcal{A}\left[\phi_p(r_1)\phi_s(r_2)\right] - \epsilon^2 \mathcal{A}\left[\phi_p(r_1)\phi_p(r_2)\right]$$

$$= 2\epsilon \mathcal{A}\left[\phi_s(r_1)\phi_p(r_2)\right]$$

C.f. Harmonic oscillator shell model

$^{12}\text{C}: (0s)^4(0p)^8$
Effective Interaction

- Finite-range effective interaction such as Gogny force

\[ v_{ij} = \sum_{k=1,2} (W_k + B_k P_\sigma - H_k P_\tau - M_k P_\sigma P_\tau) e^{-(r_i - r_j)^2/a_k^2} + t_\rho (1 + P_\sigma) \rho(r_i)^\sigma \delta(r_i - r_j) \]

\[ \langle V \rangle = \frac{1}{2} \sum_{i=1}^A \sum_{j=1}^A \sum_{k=1}^A \sum_{l=1}^A \langle ij | v|k\rangle \langle kl - lk | B_{ki}^{-1} B_{lj}^{-1} \rangle \sim A^4 \]

- Skyrme force, in recent calculations.

\[ v_{ij} = t_0 (1 + x_0 P_\sigma) \delta(r) + \frac{1}{2} t_1 (1 + x_1 P_\sigma) [\delta(r) k^2 + k^2 \delta(r)] \]

\[ + t_2 (1 + x_2 P_\sigma) k \cdot \delta(r) + t_3 (1 + x_3 P_\sigma) [\rho(r_i)]^\alpha \delta(r) \]

\[ r = r_i - r_j \]

\[ k = \frac{1}{2\hbar} (p_i - p_j) \]

\[ \langle V \rangle = \int \mathcal{V} (\rho(r), \tau(r), \Delta \rho(r), j(r)) dr \sim A^2 V \]

\[ \rho(r) = \left( \frac{2\nu}{\pi} \right)^{3/2} e^{-2\nu (r - R_{ij})^2} B_{ij} B_{ji}^{-1}, \quad R_{ij} = \frac{1}{2\sqrt{\nu}} (Z_i^* + Z_j) \]

- Less computational cost for heavy systems.
- Momentum dependence is not good at high energies.
Nuclear structure

**AMD wave function**

\[ |\Phi(Z)\rangle = \det_{ij} \left[ \exp\left\{-\nu(r_i - Z_j/\sqrt{\nu})^2\right\} \chi_{\alpha_j}(i) \right] \]

Search the energy minimum.

**Ground state nucleus as the energy minimum**

\[ i\hbar \sum_{j} C_{i\sigma,j\tau} \frac{dZ_{j\tau}}{dt} = (\lambda + i\mu) \frac{\partial H}{\partial Z_{i\sigma}} \]

\[ \Rightarrow \frac{dH}{dt} < 0 \quad (\mu < 0) \]

**Density of B-isotopes**

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \rho_p )</th>
<th>( \rho_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>13B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Binding Energy**

\( (N, Z) - 8A \) MeV

**AMD**

**Exp.**
Stochastic two-nucleon collisions

- Cross section $\frac{d\sigma_{NN}}{d\Omega}(E, \theta)$ in nuclear medium.
- Pauli blocking for the final state.
  (Almost automatic in AMD)

$$W_{i\rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta(E_f - E_i)$$

Stochastic equation of motion

$$\frac{d}{dt} Z_i = \{Z_i, H\}_{PB} + (\text{NN collisions})$$
Multifragmentation(?) in Xe + Sn Collisions

Xe + Sn central collisions at 50 MeV/u

- **AMD with NN collisions**
- **INDRA data**, Hudan et al., PRC 67 (2003)

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<tr>
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<th>AMD</th>
<th>INDRA</th>
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<td>$M(p)$</td>
<td>40.2</td>
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<td>2.5</td>
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</tr>
</tbody>
</table>

- Expansion is not sufficient.
- Too many nucleons are emitted.
Two directions of extension of AMD (maybe almost equivalent?)

Wave-packet splitting: Give fluctuation to each wave packet centroid, based on the single-particle motion.

\[ \frac{d}{dt} Z = \{Z, \mathcal{H}\}_{PB} + \text{(NN Collision)} + \text{(W.P. Splitting)} + \text{(E. Conservation)} \]


At each two-nucleon collision

\[ N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2 \]

\[ \frac{d\sigma}{d\Omega} = F_{\text{kin}} \left| \langle \phi_1' | \phi_1^+ \rangle \right|^2 \left| \langle \phi_2' | \phi_2^- \rangle \right|^2 \left( \frac{d\sigma}{d\Omega} \right)_{\text{NN} \rightarrow \text{NN}} \]

Clusters in infinite nuclear medium

Typel et al, PRC81(2010)015803

Equation for the deuteron in medium

\[
\left[ e\left(\frac{1}{2}P + p\right) + e\left(\frac{1}{2}P - p\right)\right] \tilde{\psi}(p) \\
+ \left[ 1 - f\left(\frac{1}{2}P + p\right) - f\left(\frac{1}{2}P - p\right)\right] \int \frac{dp'}{(2\pi)^3} \langle p'|\psi|p'\rangle \tilde{\psi}(p') \\
= E \tilde{\psi}(p)
\]

- **P = 0**: Clusters at rest (relative to medium)
- **T**: temperature of medium

Coupled equations for $f_n(r, p, t)$, $f_p(r, p, t)$, $f_d(r, p, t)$, $f_t(r, p, t)$, $f_h(r, p, t)$ are solved by the test particle method.

$$\frac{\partial f_n}{\partial t} + v \cdot \frac{\partial f_n}{\partial r} - \frac{\partial U_n}{\partial r} \cdot \frac{\partial f_n}{\partial p} = I_n^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_p}{\partial t} + v \cdot \frac{\partial f_p}{\partial r} - \frac{\partial U_p}{\partial r} \cdot \frac{\partial f_p}{\partial p} = I_p^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_d}{\partial t} + v \cdot \frac{\partial f_d}{\partial r} - \frac{\partial U_d}{\partial r} \cdot \frac{\partial f_d}{\partial p} = I_d^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_t}{\partial t} + v \cdot \frac{\partial f_t}{\partial r} - \frac{\partial U_t}{\partial r} \cdot \frac{\partial f_t}{\partial p} = I_t^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_h}{\partial t} + v \cdot \frac{\partial f_h}{\partial r} - \frac{\partial U_h}{\partial r} \cdot \frac{\partial f_h}{\partial p} = I_h^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$
Clusters have to be handled in a special way

### Two-body level density

**Exact Quantum Mechanics**

**Molecular Dynamics**

---

**Two-nucleon collision:**

\[
W_{i\rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta(E_f - E_i) \sum_f |\Psi_f \rangle \langle \Psi_f | = 1
\]

What is a suitable complete basis set for the final states of a two-nucleon scattering?

- A usual choice is to change only the two.
  \[
  \sum_{k_1, k_2} |\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3, 4, \ldots)\rangle \langle \varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3, 4, \ldots)|
  \]

- If a deuteron will propagate in medium, a more suitable basis will include
  \[
  |\varphi_{k_1}(1)\psi_d(2, 3)\Psi(4, \ldots)\rangle \langle \varphi_{k_1}(1)\psi_d(2, 3)\Psi(4, \ldots)| + \ldots
  \]
Cluster Formation Cross Section

\[ N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2 \]

- \( N_1, N_2 \): Colliding nucleons
- \( B_1, B_2 \): Spectator nucleons/clusters
- \( C_1, C_2 \): \( N, (2N), (3N), (4N) \) (up to \( \alpha \) cluster)

\[ \nu_{NN} d\sigma(NBNB \rightarrow CC) = |\langle \varphi'_1 | \varphi_1^+ q \rangle|^2 |\langle \varphi'_2 | \varphi_2^- q \rangle|^2 |M|^2 \delta(H - E) p_{rel}^2 dp_{rel} d\Omega \]

\[ d\sigma = F_{kin} |\langle \varphi'_1 | \varphi_1^+ q \rangle|^2 |\langle \varphi'_2 | \varphi_2^- q \rangle|^2 \left( \frac{d\sigma}{d\Omega} \right)_{NN \rightarrow NN} \]

The cross section is given from the NN cross section.

Similar to Danielewicz et al., NPA533 (1991) 712.
Construction of Final States

Clusters (in the final states) are assumed to have \((0s)^N\) configuration.

\[|\Phi^q\rangle\]

After \(p^{(0)} \rightarrow p^{(0)} + q\)

\[|\Phi'_1\rangle\]
\[|\Phi'_2\rangle\]
\[|\Phi'_3\rangle\]

\[N + B_1 \rightarrow C_1\]
\[N + B_2 \rightarrow C_2\]
\[N + B_3 \rightarrow C_3\]

Final states are not orthogonal: \(N_{ij} \equiv \langle \Phi'_i | \Phi'_j \rangle \neq \delta_{ij}\)

The probability of cluster formation with one of \(B\)'s:

\[\hat{P} = \sum_{ij} |\Phi'_i\rangle N_{ij}^{-1} \langle \Phi'_j|, \quad P = \langle \Phi^q | \hat{P} | \Phi^q \rangle \neq \sum_i |\langle \Phi'_i | \Phi^q \rangle|^2\]

\[
\begin{cases} 
P & \Rightarrow \text{Choose one of the candidates and make a cluster.} \\
1 - P & \Rightarrow \text{Don’t make a cluster (with any } n\uparrow). 
\end{cases}
\]
Effect of Clusters on the Density Evolution

Without cluster correlations (AMD with NN collisions)

With cluster correlations

During the time evolution, clusters are . . .

- formed at NN collisions.
- propagated by AMD equation. (nothing special)
- broken by NN collisions. (nothing special)
Effects of Cluster and C-C Correlations on Fragmentation

Usual NN collisions

With Clusters

With C & C-C

<table>
<thead>
<tr>
<th></th>
<th>w/o C</th>
<th>with C</th>
<th>C &amp; C-C</th>
<th>INDRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M(p) )</td>
<td>40.2</td>
<td>10.9</td>
<td>10.8</td>
<td>8.4</td>
</tr>
<tr>
<td>( M(\alpha) )</td>
<td>2.5</td>
<td>23.2</td>
<td>10.7</td>
<td>10.1</td>
</tr>
<tr>
<td>( Z_{gas}/Z_{tot} )</td>
<td>55%</td>
<td>78%</td>
<td>43%</td>
<td>(40-50%)</td>
</tr>
</tbody>
</table>

Gas = \( \sum \) (particles of \( A \leq 4 \))

Liquid = \( \sum \) (heavier fragments)
Cluster-Cluster Correlations

Relative motions between clusters should be treated quantum mechanically. In AMD,

- The binding energy a few clusters is reasonably correct,
- but the phase space of bound configuration is too small.

\[
\begin{align*}
\text{e.g. } ^7\text{Li} &= \alpha + t - 2.5 \text{ MeV} \\
|\alpha + t\rangle &\rightarrow |^7\text{Li}\rangle \text{ with probability } |\langle ^7\text{Li}|\alpha + t\rangle|^2
\end{align*}
\]

At every time step, Clusters \( C_1 \) and \( C_2 \) are bound: \( P_{\text{rel}} \rightarrow 0 \),

- if \( C_j \) is the cluster closest to \( C_i \), \((i, j) = (1, 2) \) or \((2, 1) \),
- and if they are moderately separated, \( |R_{\text{rel}}| < R_{\text{max}} \),
- and if they are moving slowly away from each other, \( |P_{\text{rel}}| < P_{\text{max}} \) and \( P_{\text{rel}} \cdot R_{\text{rel}} > 0 \).

\[
p_{\text{max}}^2/2\mu = 8 \text{ MeV}, \quad R_{\text{max}} = 5 \text{ fm} \quad \text{(adjustable)}
\]

Energy is conserved by scaling the relative momentum between the \( C_1\)-\( C_2 \) pair and a third cluster \( C_3 \).
Effect of Cluster and C-C Correlations on the Density Evolution

Without cluster correlations (AMD with NN collisions)

With cluster correlations

With cluster and cluster-cluster correlations
Au + Au Central Collisions at Higher Energies

\[ E/A = 150 \text{ MeV} \]

\[ E/A = 250 \text{ MeV} \]

\[
\begin{array}{l}
\text{with C & C-C} \\
M(p) \quad 32.8 \\
M(\alpha) \quad 20.1 \\
Z_{\text{gas}}/Z_{\text{tot}} \quad 71\% \\
\hline
\text{FOPI} \\
M(p) \quad 26.1 \\
M(\alpha) \quad 21.0 \\
Z_{\text{gas}}/Z_{\text{tot}} \quad 73\% \\
\end{array}
\]

\[
\begin{array}{l}
\text{with C & C-C} \\
M(p) \quad 42.0 \\
M(\alpha) \quad 19.4 \\
Z_{\text{gas}}/Z_{\text{tot}} \quad 80\% \\
\hline
\text{FOPI} \\
M(p) \quad 31.9 \\
M(\alpha) \quad 18.2 \\
Z_{\text{gas}}/Z_{\text{tot}} \quad 83\% \\
\end{array}
\]

FOPI data: Reisdorf et al., NPA 612 (1997) 493.

A. Ono (Tohoku U)

Light cluster production in antisymmetrized molecular dynamics

ECT* SSNHC 2014
Energy spectra of clusters

$^{124}\text{Sn} + ^{124}\text{Sn}$ and $^{112}\text{Sn} + ^{112}\text{Sn}$ central collisions at 50 MeV/nucleon

$\Rightarrow$ Energy spectra of tritons and $^3\text{He}$ emitted to transverse directions

SLy4 ($L = 46 \text{ MeV}$)  

$L = 108 \text{ MeV}$

- Triton/$^3\text{He}$ is sensitive to the symmetry energy ($L$). — Fractionation at low density.
Energy spectra of clusters

\(^{124}\text{Sn} + ^{124}\text{Sn}\) and \(^{112}\text{Sn} + ^{112}\text{Sn}\) central collisions at 50 MeV/nucleon

⇒ Energy spectra of tritons and \(^3\text{He}\) emitted to transverse directions

\[
\begin{array}{c|c|c}
\hline
Y(t)/Y(3\text{He}) & L = 46 & L = 108 \\
\hline
E/A < 10\text{ MeV} & 4.76 & 3.57 \\
E/A > 20\text{ MeV} & 2.25 & 1.65 \\
\hline
\end{array}
\]

- Triton/\(^3\text{He}\) is sensitive to the symmetry energy \((L)\). — Fractionation at low density.
Energy spectra of clusters

$^{124}\text{Sn} + ^{124}\text{Sn}$ and $^{112}\text{Sn} + ^{112}\text{Sn}$ central collisions at 50 MeV/nucleon

$\Rightarrow$ Energy spectra of tritons and $^3\text{He}$ emitted to transverse directions

Data @NSCL/MSU

Liu et al., PRC86(2012)024605

- Triton/$^3\text{He}$ is sensitive to the symmetry energy ($L$). — Fractionation at low density.
- To explain the data, the low-energy part has to be more neutron rich.
Composition of the gas part

\[ ^{124}\text{Sn} + ^{124}\text{Sn} \text{ central collisions at 50 MeV/u} \]

Light particles emitted to \( 60^\circ < \theta_{\text{cm}} < 120^\circ \)

**Velocity distribution of gas nucleons**

(in c.m. system)

**Symmetry energy**

\[ \uparrow \]

\( n/p \) yield and spectra of the gas part

\[ \uparrow \]

**Cluster correlations** should be well understood, because the \( \alpha \)-cluster formation will have a large impact.

\( \alpha \) particles are symmetric nuclei \( (N = Z) \), but the asymmetry of the system increases by the emission of \( \alpha \) particles.

\[ (N_{\text{tot}}, Z_{\text{tot}}) - n_\alpha ^4\text{He} = (N_{\text{tot}} - 2n_\alpha , \ Z_{\text{tot}} - 2n_\alpha ) \]
Two directions of extension of AMD (maybe almost equivalent?)

Wave-packet splitting: Give fluctuation to each wave packet centroid, based on the single-particle motion.

\[ \frac{d}{dt} Z = \{Z, \mathcal{H}\}_{PB} + (\text{NN Collision}) \]

\[ + (\text{W.P. Splitting}) + (\text{E. Conservation}) \]


At each two-nucleon collision

\[ N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2 \]

\[ \frac{d\sigma}{d\Omega} = F_{\text{kin}} |\langle \varphi'_1 | \varphi_1^+ q | \rangle|^2 |\langle \varphi'_2 | \varphi_2^- q | \rangle|^2 \left( \frac{d\sigma}{d\Omega} \right)_{\text{NN} \rightarrow \text{NN}} \]

Mean field + Quantum branching = AMD + Wave packet splitting

At each time step \( t_0 \), for each wave packet \( k \),

\[
t = t_0 \quad \text{or} \quad t = t_0 + \tau
\]

\[
|Z_k\rangle\langle Z_k| \quad \text{Mean field} \quad \rightarrow \quad |\psi_k\rangle\langle \psi_k| \quad \text{Branching/Decoherence}
\]

\[
\int |z\rangle\langle z| w_k(z)dz \quad \text{for} \quad k = 1, \ldots, A
\]

\[
i\hbar \frac{d}{dt} |\psi_k(t)\rangle = \hbar^{HF} |\psi_k(t)\rangle
\]

\[
\frac{\partial f_k}{\partial t} = - \frac{\partial h^{HF}}{\partial p} \cdot \frac{\partial f_k}{\partial r} + \frac{\partial h^{HF}}{\partial r} \cdot \frac{\partial f_k}{\partial p}
\]

\[
|\Phi(Z)\rangle\langle \Phi(Z)| \quad |\Psi\rangle\langle \Psi| \quad \text{Branching/Decoherence} \quad \int |\Phi(z)\rangle\langle \Phi(z)| \ w(z)dz
\]

**Coherence time \( \tau \)**

- \( \tau \rightarrow 0 \) (Strongest branching)
- \( \tau = \tau_{\text{NN-coll}} \) (Decoherence at NN collisions)
- \( \tau = \tau(\rho) \) (Density-dependent)
Multifragmentation described by AMD with Wave Packet Splitting

$^{40}\text{Ca} + ^{40}\text{Ca}$ at 35 MeV/u, $b = 0$

$^{\text{Xe}} + ^{\text{Sn}}$ at 50 MeV/u, $0 \leq b \leq 4$ fm

Experiment

![Experiment](image1)

AMD

![AMD](image2)

(Gogny force)

Soft EOS, $p$-dep U

AMD with $\tau \to 0$.

Charge distribution

![Charge distribution](image3)

$^{\text{AMD/DS}}$ & $^{\text{AMD/D}}$ & $^{\text{INDRA data}}$

AMD/D ($\tau = 0$) & AMD/DS (finite $\tau$)

Equilibrium ensembles and caloric curves

Is the equilibrium consistent with quantum statistics?

\[ \frac{dZ}{dt} = \{Z, \mathcal{H}\}_\text{PB} + \Delta Z \implies \text{Equilibrium (Statistical Properties)} \]

Many related works by: Ono & Horiuchi, Ohnishi & Randrup, Schnack & Feldmeier, Sugawa & Horiuchi, Furuta & Ono, Hasnaoui et al.

**Equilibrium Simulation**

Solve long-time evolution for given volume \( V \) and energy \( E \).

\( \implies \) Microcanonical ensemble

\( \implies (T, P) \)

\[ \frac{1}{T} = \frac{\partial S(E)}{\partial E} = \left\langle \frac{\partial S_{\text{gas}}(E_{\text{gas}})}{\partial E_{\text{gas}}} \right\rangle_E \]

\[ \approx \left\langle \frac{3}{2} \frac{N_{\text{gas}} - 1}{E_{\text{gas}}} \right\rangle_E \approx \frac{3}{2} \left\langle \frac{E_{\text{gas}}}{N_{\text{gas}}} \right\rangle_E^{-1} \]

Furuta and Ono,

PRC79 (2009) 014608;

Comparison of reaction and equilibrium

$^{40}\text{Ca} + ^{40}\text{Ca}, \frac{E}{A} = 35 \text{ MeV}, b = 0$

{\text{States at a reaction time } t} = \equiv \equiv \text{ An equilibrium ensemble } (E, V, A = 36)

half of Ca + Ca system

$t = 100 \text{ fm/c} \quad 140 \text{ fm/c} \quad 180 \text{ fm/c} \quad 300 \text{ fm/c}$

$Y(Z)$

$\langle E*/A \rangle$ [MeV]
As observed in experiments, many nucleons are bound in light clusters and heavier fragments in heavy-ion collisions.

It is not easy to explain the small number of nucleons by models based on single-nucleon motions. However, the situation can be improved very much by assuming one of

- strong decoherence to localize the single-particle wave functions.
- light clusters treated explicitly as quantum bound objects which are formed in the final states of two-nucleon collisions.

Formation and existence of light clusters influence very much the global reaction dynamics and the bulk nuclear matter properties.

The binding of several clusters to form nuclei should also be considered explicitly.

Some cluster observables, such as $t/3^\text{He}$ ratio, are sensitive to the density dependence of symmetry energy. These observables are likely to be affected by the large yield of $\alpha$ clusters.
Results of AMD with wave packet splitting

Xe + Sn at 50 MeV/u, $0 \leq b \leq 4$ fm


Fragment charge distribution

Multiplicities of nucleons and clusters

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<tr>
<td>n</td>
<td>44.3</td>
<td></td>
</tr>
<tr>
<td>p</td>
<td>25.3</td>
<td>≫ 8.4</td>
</tr>
<tr>
<td>d</td>
<td>7.5</td>
<td>4.4</td>
</tr>
<tr>
<td>t</td>
<td>1.9</td>
<td>3.3</td>
</tr>
<tr>
<td>$^3$He</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>5.0</td>
<td>≪ 10.1</td>
</tr>
</tbody>
</table>

- AMD:
  $^{112}$Sn + $^{112}$Sn, $E/A = 50$ MeV, $0 < b < 2$ fm
  $\sigma_{NN}(\rho, E)$, SLy4 (MD-corrected)

- INDRA:
  Xe + Sn, $E/A = 50$ MeV, central
  Hudan et al., PRC67 (2003) 064613.

Gogny force and a reduced $\sigma_{NN}(\rho, E)$

Coherence time: $\tau_{NN-coll}$ or $\tau \rightarrow 0$
Dependence on the way of cluster formation

$^{40}\text{Ca} + ^{40}\text{Ca}$ at 80 MeV/nucleon

$0 < b < 3$ fm

- Reduce the probabilities of $\rightarrow \text{C + C}$
  and $\rightarrow \text{N + N}$, relative to $\rightarrow \text{N + C}$.

- Change the way of selecting one of B’s.

\[ B_1(n \uparrow) \quad B_2(n \uparrow) \quad \text{N}(p \uparrow) \quad B_3(n \uparrow) \]
Energy spectra and the ratio of clusters (t and $^3$He) in $^{124}\text{Sn} + ^{124}\text{Sn}$ and $^{112}\text{Sn} + ^{112}\text{Sn}$ central collisions at 50 MeV/u ($60^\circ < \theta_{cm} < 120^\circ$).

SLy4 ($L = 46$ MeV)

$L = 108$ MeV

- t and $^3$He
- System ($N/Z$)
- Symmetry energy

M. Youngs, NuSYM13
Cluster put into a nucleus (AMD)

$\alpha$ cluster $\approx |\alpha, Z\rangle = \text{four wave packets with different spins and isospins placed at the same phase space point } Z$. (Energies are defined relative to $|^{40}\text{Ca}\rangle$.)

$$E_\alpha : \mathcal{A} |\alpha, Z\rangle |^{40}\text{Ca}\rangle$$

$$E_{\text{nucleon}} : \mathcal{A} |Z\rangle |^{40}\text{Ca}\rangle \quad (\text{nucleon} = p \uparrow, p \downarrow, n \uparrow, n \downarrow)$$

$$\Delta E_\alpha = E_\alpha - (E_{p\uparrow} + E_{p\downarrow} + E_{n\uparrow} + E_{n\downarrow})$$

- Due to the shell effect in a finite nucleus, $E_{\text{nucleon}} \approx E_F + \hbar \omega$
  $\Rightarrow$ Lower $\Delta E_\alpha$

- Due to the density dependence of the Skyrme force, the interaction between nucleons in the $\alpha$ cluster is weakened in the nucleus.
  $\Rightarrow$ Dependence on $P$ or $v_z$.

$\text{Re} \frac{Z}{\sqrt{\nu}} = (0, y, 0), \quad \frac{2\hbar \sqrt{\nu} \text{Im} Z}{M} = (0, 0, \nu_z)$
Cluster Gas and Liquid-Gas Transition

Light cluster production in antisymmetrized molecular dynamics

A. Ono (Tohoku U)

ECT* SSNHIC 2014 34 / 29
The average asymmetry and the width are sensitive to the symmetry energy.

Compared to data, $Z \geq N$ fragments are overproduced.
Fragment Isotope Distributions


\[ \langle N - Z \rangle = 0.75 \text{ and } 0.99 \]

- The average asymmetry and the width are sensitive to the symmetry energy.
- Compared to data, \( Z \geq N \) fragments are overproduced.

Data @NSCL/MSU
Liu et al.,
PRC86(2012)024605
Kinetic energies of proton-rich fragments are anomalously large.
(generalized \(^3\)He puzzle)
Possible reasons of the difference between calculation and experiment

In order to explain the experimental data, it seems both liquid and gas (low energy part) should be more neutron rich.

- Neutrons have to be more slowly expanding.
- Momentum dependence of the symmetry potential ($m_n^*$ v.s. $m_p^*$) should be studied.
- More low-velocity $\alpha$ clusters.
  - Decay from cluster-like excited states of nuclei.
    
    \[
    ^A_C^* \rightarrow \text{Be} + \alpha \\
    ^A_O^* \rightarrow ^{A-4}\text{C} + \alpha
    \]  
    (produce neutron-rich C)
Zero-point kinetic energy $= \text{Momentum width of wave packet}$

Wigner transform of one-body density matrix

$$f(r, p) = 8 \sum_{i=1}^{A} \sum_{j=1}^{A} e^{-\frac{(r-R_{ij})^2}{2\Delta x^2}} e^{-\frac{(p-P_{ij})^2}{2\Delta p^2}} B_{ij} B^{-1}_{ji}$$

$$\approx 8 \sum_{k=1}^{A} e^{-\frac{(r-R_{k})^2}{2\Delta x^2}} e^{-\frac{(p-P_{k})^2}{2\Delta p^2}}$$

where $\Delta x^2 = 1/4\nu$, $\Delta p^2 = \hbar^2/4\nu$, $R_{ij} = \frac{1}{2\sqrt{\nu}} (Z_i^* + Z_j)$, $P_{ij} = i\hbar \sqrt{\nu}(Z_i^* - Z_j)$, and $(R_k, P_k)$ are the physical coordinates.

Kinetic energy expectation value is

$$\langle K \rangle = \sum_{i=1}^{A} \sum_{j=1}^{A} \frac{P_{ij}^2}{2M} B_{ij} B^{-1}_{ji} + \frac{3\Delta p^2}{2M} A$$

$$\approx \sum_{k=1}^{A} \frac{P_{k}^2}{2M} + \frac{3\Delta p^2}{2M} A$$

AMD respects the zero-point energies except for those of the fragment center-of-mass motions.

$$K \text{ in } \mathcal{H} = \sum_{i=1}^{A} \sum_{j=1}^{A} \frac{P_{ij}^2}{2M} B_{ij} B^{-1}_{ji} + \frac{3\Delta p^2}{2M} \left( A - N_{\text{frg}}(Z) \right)$$

where $N_{\text{frg}}(Z) \in \mathbb{R}$ is the fragment number.
Excited states of a system in a box

$W(E) \approx e^{2\sqrt{aE^*}}$

Gas
(nucleons + clusters)

Liquid-gas phase transition

Volume $V = \frac{4}{3}\pi(9\text{ fm})^3$

$E^* = 28A\text{ MeV}$

$E^* = 10A\text{ MeV}$

$E^* = 4A\text{ MeV}$

$36\text{ Ar}$
Multifragmentation Reaction

Collision of two nucleus (e.g. Xe + Sn at 50 MeV/nucleon)

\[
\begin{align*}
\text{INDRA data, Hudan et al., PRC67 (2003) 064613.}
\end{align*}
\]
Multifragmentation Reaction

Collision of two nucleus (e.g. Xe + Sn at 50 MeV/nucleon)

- Microscopic dynamics of many nucleons

INDRA data, Hudan et al., PRC67 (2003) 064613.
Multifragmentation Reaction

Collision of two nucleus (e.g. Xe + Sn at 50 MeV/nucleon)

- Microscopic dynamics of many nucleons
- Nuclear matter properties
  - Density: $\rho_0 \rightarrow 1.5\rho_0 
  \rightarrow 0.5\rho_0 \rightarrow 0$
  - Excitation energy: 12.5 MeV/nucleon ($\approx$ B.E.)
    $\rightarrow 2$ MeV/nucleon $\rightarrow 0$
  - Density fluctuation and/or cluster correlations

INDRA data, Hudan et al., PRC67 (2003) 064613.
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### Partitioning of protons
- $p \approx 10\%$
- $\alpha \approx 20\%$
- $d, t, ^3\text{He} \approx 10\%$
- $A > 4 \approx 60\%$

Exp. data (INDRA etc.)