PROGRESS IN THE THEORY OF NUCLEAR COLLECTIVE EXCITATIONS

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September 2015
Rare Isotope Science Project (RISP)

Aim: to build a heavy-ion accelerator for nuclear physics research
Plus: materials science, biomedical science

approx. ~ 2020

**Exotic nuclei, superheavy nuclei**

**Nuclear Astrophysics**
Vibrations and restoration mechanisms

Spring constant

Nuclear incompressibility?

Surface tension

Symmetry energy?

Excitation Energy (MeV)
Nuclear response theory – my progress in...

...Understanding

• New insights and new phenomena: making sense of the electric-dipole spectrum
• Pygmy resonances and the surface dipole mode (or IS-LED)
• Relevant for:
  • nuclear reactions
  • nucleosynthesis
  • equation of state
  • neutron stars

...Methodology

• Linear response theory with microscopic (“realistic”) interactions
• The NNN in-medium force and how to implement it
• Resolving, e.g.:
  • terms that are difficult to constrain (tensor)
  • optimal density dependence?
  • effective mass?
  • controlled extensions
Overview

...Understanding

- About the E1 spectrum
- “Pygmy resonance” =
  - IS-LED
  - + 1ℏω
  - + skin oscillation?
  - + α-cluster mode
- Importance of:
  - shell effects
  - particle emission threshold

Theoretical results from QRPA+D1S model + many many data

...Methodology

- Promising results with SRPA and UCOM... (~2010)
- ...And the importance of NNN terms
- Two strategies
  - simple phenomenological contact term
    - ... and some interesting lessons
  - chiral NNN + SRG **new**
1. NEW(?) DIPOLE MODES IN STABLE AND EXOTIC NUCLEI

“Pygmy resonance” = IS-LED + 1ℏω + skin oscillation? + α-cluster mode

Importance of:
- shell effects
- particle emission threshold
New (?) dipole modes in stable and exotic nuclei


Typical (schematic) dipole spectrum

E1 strength below / around threshold

(below ~ 8-10 MeV)

Related reviews:
- D.Savran, T.Aumann, A.Zilges, Prog.Part.Nucl.Phys. 70, 210 (2013)
An elegant early interpretation: $1\hbar\omega$ transitions

Asymmetries (differences) in proton and neutron particle-hole energies...

... leads to remainders of E1 strength in the $1\hbar\omega$ region, below the GDR – *non collective*

A.Oros et al., PRC57,990(1998)
“Isospin splitting” in stable nuclei

... in N=82 isotones, [D.Savran et al.] as well as:

Crespi et al., PRC91,024323(2015) - LNL
Endres et al., PRL105,212503(2010) - KVI
Crespi et al., PRL113,012501(2014) - LNL
"Isospin splitting" in stable nuclei

- $(\gamma,\gamma')$: Isovector (E1) response
- <1% TRK

- $(\alpha,\alpha'\gamma)$: Isoscalar segment on the lower end
- up to ~10% of EWSR

- IS-Collective – What is it?
“Isospin splitting” in stable nuclei

- \((\gamma,\gamma')\): Isovector (E1) response
  - <1% TRK
- \((\alpha,\alpha'\gamma)\): Isoscalar segment on the lower end
  - up to \(\sim 10\%\) of EWSR

**Interpretation I:** neutron-skin oscillation (responds to the isoscalar probe)

**Objection 1:** then IV strength several times too large
**Objection 2:** Similar IS dipole strength is observed in symmetric nuclei too

**Interpretation II:** elementary IS surface vibration

[Image « stolen » from Bastrukov et al., PLB664, 258]
48Ca vs. 40Ca

strong isoscalar state in 48Ca with very little E1 strength, just like 40Ca - no neutron-skin mode
Also predicted...

- Sn isotopic chain: up to $^{132}$Sn
  - much softer beyond shell closure

- Ni isotopic chain: beyond $^{62}$Ni, bimodal structure due to coupling with surface neutrons

- And many other spherical nuclei ($^{208}$Pb, Mo, ...)

- Both calculated and observed: $^{16}$O, $^{40,48}$Ca, $^{208}$Pb, ...
Energy and structure

Fig. 1. (a) The energy (approximately depicted) of the observed low-energy isoscalar $1^-$ transitions in $^{12}$C [11], $^{16}$O [12, 13], $^{40}$Ca [2, 14], $^{48}$Ca [5], $^{58}$Ni, $^{90}$Zr [2, 14], $^{94}$Mo [4], $^{124}$Sn, $^{138}$Ba, $^{140}$Ce [3], $^{208}$Pb [2, 6] in comparison with the energy of the first excited state $30A^{-1/3}$ MeV [2], and the empirical 1$^-$ energy with the Gogny D1S interaction; and with the excitation of the energy-weighted sum rule with $\rho_0 = 0.16$ fm$^{-3}$, $R = 2.634$ fm (r.m.s.) $R_{\text{rms},^{16}\text{O}} \approx 2.7$ fm. The RPA result for $^{132}$Sn is also shown, for the coordinate shown on the top axis $r(\text{Sn}) = r(\text{O}) \times R_{\text{rms},^{132}\text{Sn}} / R_{\text{rms},^{16}\text{O}}$ (point-proton r.m.s. radii as calculated with the Gogny D1S interaction).

- Universal feature of “ordinary” nuclei
- No “skin” origin
- A few % of the IS EWSR = very strong!
- “Surface dipole mode”
Collective model

If the isoscalar dipole energy weighted sum rule is exhausted by one state, then the transition density of this state can be expressed as

\[
\rho^{(1)}(r) = -\frac{\beta_1}{R\sqrt{3}} \left[ 3r^2 \frac{d}{dr} + 10r - \frac{5}{3} r^2 \frac{d}{dr} \right. \\
\left. + \epsilon \left( r \frac{d^2}{dr^2} + 4 \frac{d}{dr} \right) \right] \rho_0(r), \tag{4}
\]

where

\[
\beta_1^2 = \frac{6\pi \hbar^2}{mA E_x} \frac{R^2}{(11 \langle r^4 \rangle - \frac{25}{3} \langle r^2 \rangle^2 - 10\epsilon \langle r^2 \rangle)}, \tag{5}
\]

and \(\beta_1\) is the collective coupling parameter for the isoscalar dipole resonance and \(R\) is the half-density radius of the Fermi mass distribution.
Low-energy vibration vs giant resonance

isoscalar dipole channel

Low-energy IS dipole transition vs compression dipole resonance

Transition density: node
Velocity fields: toroidal vs. compressional

PP, VYuPonomarev, RRoth, JWambach, EPJA47,14(2011)
Low-energy vibration vs giant resonance

Typical RPA result for $2^+$ spectrum:
• low-energy vibration
• Giant resonance

Transition density: surface-peaked
Velocity fields: toroidal vs. hydrodynamical

Analogy in the dipole spectrum?

S.Péru et al., EPJA26,25(2005)

PP, PhD thesis (for $^{56}$Ni)
Relation to clustering?

- Dipole transitions because of alpha clustering: F.Iachello 1985

M. Spieker et al. PRL114, 192504 (2015) Clustering in Rare Earths (IBM model)

To be continued!
What about the purely IV segment?

consistent with single-particle transitions [QRPA]

...etc (stable Sn, Ni, Ca)

More-recent studies corroborate such an interpretation [Baran et al., Nazarewicz&Reinhard, ... ]
An elegant early interpretation: $1\hbar\omega$ transitions

Asymmetries (differences) in proton and neutron particle-hole energies...

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Two more things:

• Role of shell effects
  • Polarizability and neutron skin
  • Onset of soft modes

• Loosely bound nuclei
  • Exotic effects owing to low emission threshold
  • (Must treat properly the extended wavefunctions)
Polarizability vs skin

Reminder:
The neutron-skin thickness and the dipole polarizability are correlated ... but the correlation is model-dependent (cf Skyrme vs. RMF) 

W.Nazarewicz, P.-G.Reinhard, et al ...

\[ a_D = \frac{\hbar c}{2\pi^2} \int \frac{\sigma_{abs}}{\omega^2} d\omega \]

Reference value: assuming 100% GDR

\[ a_{D_{coll}} = \frac{\hbar c}{2\pi^2} \frac{60NZMeVmb}{AE_{GDR}^2} \]

\[ E_{GDR} = (31.2A^{-1/3} + 20.6A^{-1/6}) \text{ MeV} \]

Measurement: D.Rossi et al., PLR 111,242503
Role of shell structure

“Soft” modes beyond...

- N=28 (Ca) [PLB,2014]
- N=82 (Sn) [PRC,2014]

(see also: Inakura et al.)
Proton-rich nuclei and their stable mirrors

Disentangling EoS properties from structure effects?

Summarizing part I:

- Nothing mysterious about the IS segment of the “pygmy dipole strength” in ordinary nuclei:
  - Surface dipole mode is a universal feature
  - $\alpha$-Clustering may occur too (cf. rare earths)
- E1 strength is consistent with single-particle transitions
- Is there a “neutron-skin” mode? Maybe, ...
  - in very exotic nuclei (beyond certain shell closures) OR
  - very close to the particle threshold  
    cf Ni isotopes PP et al., PRC (2015)
  - (Asy-stiff functionals predict it in stable nuclei)

- Need measurements on either side of shell closure, and of the emission threshold, around 9-10MeV
- Challenging work

Awaiting new, complete spectra!
2. NUCLEAR RESPONSE WITH REALISTIC INTERACTIONS

Promising results with SRPA and UCOM... (~2010)
...and the importance of NNN
First applications with chiral + SRG
Related papers...

- Roth, Hergert, PP, Neff, Feldmeier, PRC72,034002(2005) *Matrix elements and few-body calculations*
- Roth, PP, Paar, Hergert, Neff, Felmeier, PRC73,044312(2006) *Hartree-Fock and perturbation theory*
- Paar, PP, Roth, PRC74,014318(2006) *RPA*
- PP, Roth, Paar, PRC75,014310(2007) *RPA with explicit ground-state correlations*
- PP, Roth, PLB671,356 (2009) *Second RPA*
- PP, Roth, PRC81,024317(2010) *Second RPA*
- H. Hergert, PP, R. Roth, PRC83,064317 *QRPA (with SRG)*
- Usman, ..., PP, ... PLB698(2011)191 *Second RPA and the GQR of $^{40}$Ca in $(p,p')$*
- A. Günther, PP, R. Roth, JPG41,115107(2014) *RPA (with SRG)*
- R. Trippel, PP, R. Roth, in preparation *SRPA with $\chi(\text{NN+NNN}), \text{SRG}$*
In a nutshell

- Start from a realistic, microscopic NN(+NNN) potential
  - Precise fits (Argonne, Bonn)
  - Chiral
- Soften it by a unitary transformation
  - UCOM: state-independent SRC
  - SRG: ~diagonal Hamiltonian in momentum space
- Choose a suitable many-body method
  - Linear response theory -> RPA
- Examine convergence w.r.t. model space
  - -> SecondRPA

The fine print:

- Truncations at the 2N level (nowadays 3N)
- There is ~one adjustable parameter, chosen to minimize residual 3N effects (exact calculations of energies in few-body systems)
Unitary correlation operator method

• Explicit correlations by means of unitary operators imprinted in the wavefunction or the operators

\[ |\tilde{\Psi}\rangle = C_\Omega C_r |\Psi\rangle = U |\Psi\rangle \quad \Rightarrow \quad \langle \tilde{\Psi} | A |\tilde{\Psi}\rangle = \langle \Psi | \tilde{A} |\Psi\rangle \]

• A : e.g. Hamiltonian \( \Rightarrow \) \( H_{\text{eff}} \)
• U determined variationally

In practice: truncate at 2N nucleon level and adjust range of tensor correlator using exact calculations

\( \Rightarrow \) One parameter takes care of genuine and induced 3N

Only state-independent short-range correlations!
Long-range ones to be taken care of by the model space
Similarity renormalization group

• Flow equations

\[ \frac{d\tilde{A}_\alpha}{d\alpha} = [\eta_\alpha, \tilde{A}_\alpha] \]

• Towards diagonal Hamiltonian in momentum space: KE as generator

\[ \eta_\alpha = (2\mu)^2 [T_{\text{int}}, \tilde{H}_\alpha] \]

• Also unitary transformation

\[ \tilde{A}_\alpha = U_\alpha^\dagger A U_\alpha \]

**In practice:** adjust flow parameter, e.g. using exact calculations; possibly 3N force for saturation

one ~ two parameters

... in both cases: In first applications, truncation at 2N level; ~1 parameter, chosen to eliminate the effect of 3N forces... from the **masses** of few-body systems

(UCOM: tensor correlator range; SRG: flow parameter)
AV18-UCOM-Hartree-Fock + perturbative corrections

\[ E^{(2)} = -\frac{1}{4} \sum_{i,j} \sum_{a,b} \frac{|\langle ij | H_{\text{int}} | ab \rangle|^2}{e_a + e_b - e_i - e_j} \]

\[ H_{\text{int}} = T_{\text{rel}} + V_{\text{UCOM}} \]

3N force

Roth, PP, Paar, Hergert, Neff, Felmeier, PRC73, 044312 (2006)
The first "self-consistent", large-scale HF-SRPA calculations:

- Full space
- No divergencies
- No double counting
- Computing and algebraic techniques
SRPA and UCOM

3N force?
Exploring two strategies:

1. A phenomenological NNN contact term (equivalent to a density-dependent NN term)
   - Results with (Q)RPA + SRG-evolved AV18
   - See how far we can go with it
   - Rich NN interaction, giving bound nuclei – NNN a rather weak correction

2. (S)RPA with normal-ordered realistic NNN \( \rightarrow \) work with two-body formalism
1) Phenomenological 3N contact term

A. Günther, PP, R. Roth, JPG41, 115107(2014)
QRPA applications: H. Hergert, PP, R. Roth, PRC83, 064317

Requirement: correct radii GRs:
Mostly within 20% from other results
Bound nuclei at the 2N level
DDI: up to an order of magnitude smaller than « usual »

\[ V_{3N} = C_{3N} \delta^{(3)}(\bar{r}_1 - \bar{r}_2)\delta^{(3)}(\bar{r}_1 - \bar{r}_3) \]
Strength of in-medium 3N force?

- **Phenomenological** functionals: a density-dependent term is required for saturation.
- Without it, the ground state collapses to large density.
- In the **present approaches**, most nuclei “saturate” already at the 2N level; the 3N only improves the saturation point.

- **3N force an order of magnitude weaker than a typical density-dependent interaction**, e.g.:

  \[
  t_3 \text{(UCOM-SRG)} \times \rho_0 \div t_3 \times \rho_0^\alpha \text{(D1S)} \sim 0.08
  \]

- A handle on the “acceptable” strength of a (in-medium) DDI vs. momentum dependence etc.?  

*H. Hergert, PP, R. Roth, PRC83, 064317; A. Günther, PP, R. Roth, JPG41, 115107(2014);*
2) SRG-evolved chiral interactions+NO
Normal-ordered 3N force:

- Rewrite the 3N interaction in normal-ordered form:

\[
\hat{V}_{3N} = \frac{1}{36} \sum_{ijk,i'j'k'} v_{ijk,i'j'k'}^{3N} \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k^\dagger \hat{a}_{i'} \hat{a}_{j'} \hat{a}_{k'} = N[\hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k^\dagger \hat{a}_{i'} \hat{a}_{j'} \hat{a}_{k'}] \\
+ \sum_{1 \text{ contr.}} N[\hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_{i'} \hat{a}_{j'} \hat{a}_{k'}] \\
+ \sum_{2 \text{ contr.}} N[\hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_{i'} \hat{a}_{j'} \hat{a}_{k'}] \\
+ \sum_{3 \text{ contr.}} N[\hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_{i'} \hat{a}_{j'} \hat{a}_{k'}]
\]

- HF with full interaction
- Normal ordering, neglect last term
- (S)RPA without it \(\Rightarrow\) 2N formalism \(\Rightarrow\) numerical simplification

R.Roth et al., PRL109,052501

R.Trippel, PP, R.Roth, in preparation
NNN and normal-ordering in (S)RPA

NN (N3LO) + NNN (N2LO)
SRG evolution: up to induced NNN terms
Normal ordering: Work with a NN formalism
Weak residual 3N force

☑ First application of chiral NN+NNN to giant resonances
☑ Remaining: convergence issues

Compare with UCOM:
• similar shift; stronger fragmentation??
Summary of Part II

• Nuclear response with microscopic NN + NNN interactions: It’s possible, especially with normal ordering
• Because the “in-medium” NNN is relatively small

• SRPA with NN+NNN: promising first results
  • To be tackled: restore the stability condition, violated in SRPA [P.P., PRC90,024305]; via GSC or subtraction procedure

• Towards a predictive theory of nuclear response

• Insight and guidance for improvements of phenomenological effective interactions
Overview

...Understanding

- About the E1 spectrum
- “Pygmy resonance” = IS-LED
- $+ 1\hbar \omega$
- + skin oscillation?
- + $\alpha$-cluster mode

Importance of:

...Methodology

- Promising results with SRPA and UCOM... (~2010)
- And the importance of NNN terms
- Two strategies
  - simple phenomenological contact term
  - ... and some interesting lessons
- chiral NNN + SRG **new**

Thank you!
감사합니다!
Several RPA variants

- RPA in configuration space
  - Most basic: Choose a Hamiltonian and a basis (H.O.), solve Hartree-Fock, build RPA matrix using the same Hamiltonian
- QRPA in configuration space
  - Similar: Begin with Hartree-Fock-Bogolyubov
  - Same Hamiltonian in all channels
  
  These take care of Landau damping
- RPA in coordinate space:
  - Convergence guaranteed
  - With proper boundary conditions, you can get the escape width
- Second RPA:
  - Includes 2p2h configurations -> collisional damping
- Renormalized RPA, ... PVC, ...
Ground-state vs. excited state properties

\[ \frac{E}{A}(\rho, \delta^2) = \left( \frac{E}{A} \right)_{\rho=\rho_0} + \frac{(\rho - \rho_0)^2}{18} \left( \frac{\partial^2 (E/A)}{\partial \rho^2} \right)_{\rho=\rho_0} + \frac{\delta^2}{2} \left( \frac{\partial^2 (E/A)}{\partial \delta^2} \right)_{\delta=0} \]

Absolute values vs. derivatives
Role of shell structure

“Soft” modes beyond...

- N=28 (Ca) [PLB, 2014]
- N=82 (Sn) [PRC, 2014]

(see also: Inakura et al.)
Electric dipole spectrum of $^{208}$Pb

1. Tamii et al., PRL107(2011)062502
2. Crespi et al., PRL 113(2014)012501
3. Poelhakken et al., PLB278(1992)423
4. Poelhakken et al., PLB278(1992)423
3. Ryezayeva et al., PRL89 (2002) 272502
4. GDR (here: Tamii et al.)