Hadron resonance gas (HRG) model can describe the QCD EoS as well as the 2\textsuperscript{nd} order fluctuations of conserved charge reasonably well at zero net baryon density.

How about higher order fluctuations and EoS at non-zero baryon density? How important are the repulsive baryon-baryon interactions?

In this talk:

1) The virial expansion in the nucleon gas
2) Fluctuations and correlations of conserved charges HRG with repulsive mean-field
3) EoS at non-zero density

in collaboration with P. Huovinen, arXiv:1708.00879, work in progress
Higher order fluctuations of conserved charges in T>0 QCD

\[ \chi_n^X = T^n \frac{\partial^n (p(T,\mu_X)/T^4)}{\partial \mu_X^n} \bigg|_{\mu_X=0}, \quad \chi^{XY}_{nm} = T^{n+m} \frac{\partial^{n+m} (p(T,\mu_X,\mu_Y)/T^4)}{\partial \mu_X^n \partial \mu_Y^m} \bigg|_{\mu_X=0,\mu_Y=0}. \]

Bazavov et al, PRL 111 (2013) 082301

The above combinations should be 0 or 1 in HRG independent of details of hadron spectrum and HRG description breaks down close to the transition temperature (even below \( T_c \)).
Virial expansion in the nucleon gas

\[ p = p^{\text{ideal}} + T \sum_{ij} b^i_j(T) e^{\beta \mu_i} e^{\beta \mu_j} \]

\( b^i_j \) can be related to the S-matrix of scattering of particles \( i \) and \( j \)

\( \pi \pi, KK, \pi N \) and \( NK \) scattering are dominated by resonances:

\[ p \rightarrow p^{\text{ideal}}_{\pi K N} + p^{\text{ideal}}_{\text{resonances}} \]

No resonances in \( NN \) interactions

Gas of nucleons:

\[ p(T, \mu) = p_0(T) \cosh(\beta \mu) + 2b_2(T)T \cosh(2\beta \mu) \]

\[ p_0(T) = \frac{4M^2T^2}{\pi^2} K_2(\beta M) \]

\[ b_2(T) = \frac{2T}{\pi^3} \int_0^\infty dE (\frac{ME}{2} + M^2) K_2 \left( 2\beta \sqrt{\frac{ME}{2} + M^2} \right) \frac{1}{4i} \text{Tr} \left[ S^\dagger \frac{dS}{dE} - \frac{dS^\dagger}{dE} S \right] , \]

factorization in \( \mu \) and \( T \) dependent part is broken

HRG model

Dashen, Ma, Berstein, PR 187 (1969) 345
Prakash, Venugopalan, NPA 546 (1992) 718
Virial expansion in the nucleon gas (cont’d)

Only the elastic part of the S-matrix is known

\[ \frac{1}{4i} \text{Tr} \left[ S^\dagger \frac{dS}{dE} - \frac{dS^\dagger}{dE} S \right] \rightarrow \sum_{s=\pm} \sum_J (2J + 1) \left( \frac{d\delta^{J,I=0}}{dE} + 3 \frac{d\delta^{J,I=1}}{dE} \right) \]

Use recent partial wave analysis results for $NN$ scattering (SM16, SP07)
Use effective range expansion for $E < 1 \text{ MeV}$

Workman et al, PRC 94 (2016) 065203
Arndt et al, PRC 76 (2007) 025209

\[ \frac{d}{dE} \sum_{J,s} (2J + 1) \delta^{J,I=0}_s \Rightarrow b_2(T) < 0 \]
Assume that the repulsive interactions reduce the single nucleon energies by $U = Kn_b$, where $n_b$ is the single nucleon density

\[ K \sim \int d^3r V_{NN}(r) \Rightarrow K > 0 \]

Nucleon and anti-nucleon densities

\[
\begin{align*}
n_b &= 4 \int \frac{d^3p}{(2\pi)^3} e^{-\beta(E_p - \mu + U)}, \quad \bar{n}_b = 4 \int \frac{d^3p}{(2\pi)^3} e^{-\beta(E_p + \mu + \bar{U})}, \quad E_p^2 = p^2 + M^2 \\
\partial p/\partial \mu &= n_b - \bar{n}_b \Rightarrow p(T, \mu) = T(n_b + \bar{n}_b) + \frac{K}{2}(n_b^2 + \bar{n}_b^2)
\end{align*}
\]

Small (zero) $\mu \Rightarrow \beta Kn_b \ll 1$ and

\[
\begin{align*}
n_b &\simeq n_b^0(1 - \beta Kn_b^0), \quad \bar{n}_b \simeq \bar{n}_b^0(1 - \beta \bar{K}\bar{n}_b^0) \Rightarrow \\
p(T, \mu) &= T(n_b^0 + \bar{n}_b^0) - \frac{K}{2} \left( (n_b^0)^2 + (\bar{n}_b^0)^2 \right)
\end{align*}
\]

or

\[
p(T, \mu) = p_0(T)(\cosh(\beta \mu) - \frac{KM^2}{\pi^2}K_2(\beta M) \cosh(2\beta \mu))
\]
Comparison of repulsive mean field and virial expansion

Repulsive mean field

\[ p(T, \mu) = p_0(T) \times \]
\[ \left( \cosh(\beta \mu) - \frac{KM^2}{\pi^2} K_2(\beta M) \cosh(2\beta \mu) \right) \]

2nd order virial expansion

\[ p(T, \mu) = p_0(T) \times \]
\[ \left( \cosh(\beta \mu) + \bar{b}_2(T) K_2(\beta M) \cosh(2\beta \mu) \right) \]
\[ \bar{b}_2(T) = \frac{2Tb_2(T)}{p_0(T)K_2(\beta M)} \]

In-elastic interactions become important for \( E > 400 \) MeV

\( \Rightarrow \) use \( \sigma_{el}/\sigma_{tot} \)

to estimate the uncertainties in \( b_2(T) \) due to these effects

\[ - \frac{KM^2}{\pi^2} \]

for typical phenomenological value \( K = 450 \) MeV fm\(^3\)

Sollfrank et al, PRC 55 (1997) 392
Hadron resonance gas with repulsive mean field

\[ n_B(T, \mu_B, \mu_S, \mu_Q) = \frac{T}{2\pi^2} \sum_i g_i M_i^2 K_2(\beta M_i) e^{\beta \mu_i,\text{eff}}, \quad \mu_i,\text{eff} = \sum_j q^j_i \mu_j - K n_B \]

\[ \bar{n}_B(T, \mu_B, \mu_S, \mu_Q) = \frac{T}{2\pi^2} \sum_i g_i M_i^2 K_2(\beta M_i) e^{\beta \bar{\mu}_i,\text{eff}}, \quad \bar{\mu}_i,\text{eff} = -\sum_j q^j_i \mu_j - K \bar{n}_B \]

\[ (q^1_i, q^2_i, q^3_i) = (B_i, S_i, Q_i) \]

strange and non-strange baryons interact the same way

\[ \partial p/\partial \mu_B = n_B - \bar{n}_B \text{ and leading order expansion in } \beta K n_B \Rightarrow \]

\[ p_B(T, \mu_B, \mu_S, \mu_Q) = T(n_B^0 + \bar{n}_B^0) - \frac{K}{2} \left( (n_B^0)^2 + (\bar{n}_B^0)^2 \right) \]

\[ \chi_n^B = \chi_n^{B(0)} - 2^n \beta^4 K \left( N_B^0 \right)^2, \quad (n \text{ even}) \]

\[ \chi_n^{BS} = \chi_n^{BS(0)} + 2^{n+1} \beta^5 K N_B^0 (p_B^{S1} + 2p_B^{S2} + 3p_B^{S3}) \quad (n \text{ odd}) \]

\[ N_B^0(T) = \frac{T}{2\pi^2} \sum_i g_i M_i^2 K_2(\beta M_i) \]
Comparison with lattice QCD results

Assume that only ground state baryons (octet + decuplet) contribute to $n_B$; higher resonances are treated as free particles.

Repulsive mean field calculations can explain the differences between certain higher order fluctuations and correlations; $v_2$ is not described by this simple model.
There are many more states that predicted by LQCD and quark models but are included in PDG. Missing states are important for the fluctuations of conserved charges. Bazavov, PRL 113 (2014) 072001
Missing states and the trace anomaly

- HRG is below the lattice data
Missing states and the trace anomaly

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- HRG with missing states (HRG+) describes the data better
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- Repulsive mean-field slightly reduces the HRG result
Baryon number fluctuations: missing states and repulsive mean field

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Baryon strangeness correlations: missing states and repulsive mean field

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Baryon strangeness correlations: missing states and repulsive mean field

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- HRG with missing states (HRG+) slightly over predicts the lattice data
- Repulsive mean-field reduces the HRG+ result
Pressure with repulsive mean field

Use expanded expressions (in $K n_B$) to calculate the pressure

$\mu_S = 0$

$\mu_B = 0$
$\mu_B = 100$
$\mu_B = 300$
$\mu_B = 400$

$\mu_B = 0$
$\mu_B = 100$
$\mu_B = 300$
$\mu_B = 400$

Virial expansion works only for baryon chemical potential $< 400$ MeV

The repulsive mean field reduces the pressure up 24%

For the strangeness neutral case the effects of the repulsive interactions are smaller.
Repulsive mean field reduces the energy density up to 30%.

For the strangeness neutral case the effects of the repulsive interactions are smaller.
Assume that freeze-out happens at energy density of 330 MeV/fm$^3$

Strangeness neutrality and repulsive interaction reduce the curvature of the freeze-out temperature

Strangeness neutrality and repulsive interactions reduce the net baryon density at the freeze-out
The curvature of the freeze-out line corresponding to constant energy density \(\sim 330 \text{ MeV/fm}^3\) calculated in HRG model with repulsive interactions agrees with lattice result of Bazavov et al, PRD 95 (2017) 054504
Summary

• Repulsive baryon-baryon interactions are important for baryon number fluctuations and baryon strangeness correlations as well as for EoS at non-zero baryon density, their effect could be similar to the effect of missing states in terms of size but in opposite direction.

• Mean field approach is very similar to the virial expansion in the low density regime => constraints on the mean field values.

• The simplest mean field approach can describe the differences between second and higher order baryon number fluctuations as well as baryon strangeness correlations, but certain baryon-strangeness correlations cannot be described by this simple model.

• The virial expansion is applicable for baryon chemical potential < 400 MeV.

• Future: separate treatment of strange non-strange baryons, including the effect of repulsive interactions on resonances.