Isovector observables in nuclear mean-field theories

P.–G. Reinhard

Institut für Theoretische Physik II
Universität Erlangen-Nürnberg

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Acknowledgements

Collaborators:
J. Erler, W. Nazarewicz, M. Stoitsov(†) UT Knoxville
J. Piekarewicz FSU Tallahassee
T. Niksic, N. Paar, D. Vretenar Zagreb
B. Agrawal Saha Institute Kolkatta
G. Colo, X. Roca-Maza Milano
W. Satula Warsaw
M. Kortelainen Jyväskyla
J. A. Maruhn, B. Schütrumpf Frankfurt
W. Kleinig, V. Nesterenko JINR Dubna
I. Kvasil, P. Vesely Prague

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1. Parameters and observables

2. Optimization of model parameters and subsequent variation

3. The influence of $J = a_{\text{sym}}$ on observables

4. The weak-charge formfactor and correlation with $r_n$

5. A few words about low-lying dipole strength ("pygmy region")
Exploring correlations: SHF/RMF are adjusted to data by $\chi^2$ fits. We exploit methods of error propagation in $\chi^2$ technique.

Observables: We concentrate on static isovector observables $\leftrightarrow$ symmetry energy $a_{\text{sym}} \equiv J_T$.

Trends with $J_T$: SHF and RMF-PC have similar trends, but can differ in offset $\leftrightarrow$ relativistic effect? RMF-DD has even different trends $\leftrightarrow$ much different density dependence?

Correlation with $J_T$: Group of highly coorrelated (static) isovector observables: polarizability $\alpha_D$, neutron radius $r_n$, skin $r_n - r_p$, weak-charge formfactor $F_W(q_{\text{PREX}})$.

Weak-charge formfactor $F_W$: $\chi^2$ uncertainties depend on $q_{\text{PREX}}$, maximal at $q_{\text{PREX}}$ $\leftrightarrow$ PREX data most informative.

Low $E_d$ dipole strenth: Dominated by isoscalar $L = 1$ modes, spread over several distinct modes.

Integrated dipole strength yields combined information on $a_{\text{sym}}$ and $\kappa$. 

Conclusions

- **Exploring correlations:**
  SHF/RMF are adjusted to data by $\chi^2$ fits. We exploit methods of error propagation in $\chi^2$ technique.
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- **Observables:**
  We concentrate on static isovector observables $\longleftrightarrow$ symmetry energy $a_{\text{sym}} \equiv J$

  - Trends with $J$:
    - SHF and RMF-PC have similar trends, but can differ in offset $\leftrightarrow$ relativistic effect?
    - RMF-DD has even different trends $\leftrightarrow$ much different density dependence?

  - Correlation with $J$:
    - Group of highly correlated (static) isovector observables $\longleftrightarrow$ polarizability $\alpha$, neutron radius $r_n$, skin $r_n - r_p$, weak-charge formfactor $F_W(q_{\text{PREX}})$

  - Weak-charge formfactor $F_W$:
    - $\chi^2$ uncertainties depend on $q$, maximal at $q_{\text{PREX}}$ $\leftrightarrow$ PREX data most informative

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Parameters and observables
The nuclear matter parameters (NMP)

given \( E/A(\rho) = \) energy per particle in symmetric nuclear matter (function of density \( \rho \))
this allows to define basic properties near equilibrium:

<table>
<thead>
<tr>
<th>Parameter</th>
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</tr>
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<tbody>
<tr>
<td>( E/A_{eq} )</td>
<td>binding energy per particle at equilibrium point</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>equilibrium density</td>
</tr>
<tr>
<td>( K = 9\rho_0^2 \frac{\partial^2 E}{\partial \rho^2} \frac{A}{E} )</td>
<td>incompressibility (isoscalar static response)</td>
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<tr>
<td>( \frac{m^*}{m} )</td>
<td>effective mass (isoscalar dynamic response)</td>
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<tr>
<td>( J = a_{sym} )</td>
<td>symmetry energy (isovector static response)</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>TRK sum rule enhancement ( \leftrightarrow ) isovector ( \frac{m_{1}^*}{m} ) (dynamic response)</td>
</tr>
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<td>( L = 3\rho_0 \frac{\partial \rho}{\partial \rho} J )</td>
<td>slope of symmetry energy</td>
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given \( E/A(\rho) = \) energy per particle in symmetric nuclear matter (function of density \( \rho \))
this allows to define basic properties near equilibrium:

- \( E/A_{\text{eq}} \): binding energy per particle at equilibrium point
- \( \rho_0 \): equilibrium density
- \( K = 9\rho_0^2 \partial^2 \frac{E}{A} \): incompressibility (isoscalar static response)
- \( \frac{m^*}{m} \): effective mass (isoscalar dynamic response)
- \( J = a_{\text{sym}} \): symmetry energy (isovector static response)
- \( \kappa \): TRK sum rule enhancement \( \leftrightarrow \) isovector \( \frac{m^*_1}{m} \) (dynamic response)
- \( L = 3\rho_0 \partial \rho J \): slope of symmetry energy

considered here as part of the model parameters
(e.g.: equivalent to \( t_0, x_0, t_1, x_1, t_3, x_3, \alpha \) in case of SHF)
### Observables in the pool of fit data

<table>
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<tr>
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<tr>
<td>$E_B$</td>
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</tr>
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<td>$r_C$</td>
<td>charge r.m.s. radii</td>
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Data have been scrutinized to have negligible effects from ground state correlation.
“Predicted” observables in correlation study

- neutron r.m.s. radius \( r_n \) in \(^{208}\text{Pb} \)
- neutron skin \( r_n - r_p \) in \(^{208}\text{Pb} \)
- weak-charge formfactor \( F_W(q_{\text{PREX}} = 0.475/\text{fm}) \) in \(^{208}\text{Pb} \) (related to PREX)
  \[
  F_W(q) = G_n^Z(q)F_n(q) + G_p^Z(q)F_p(q)
  \]
  \( G_{p/n}^Z = \text{weak-charge formfactor for nucleon} \)
  \( F_{p/n} = \text{formfactor of nuclear distributions (Fourier transform of } \rho(r) \text{)} \)
- dipole polarizability \( \alpha_D = \int_0^\infty dE \sigma_{\text{photoabs}}(E) E^{-2} \) in \(^{208}\text{Pb} \)
- binding energy \( E_B \) for extremely exotic nuclei (super heavy, very neutron rich)
- \( Q_\alpha \) value for super-heavy element
- \( B_f = \text{fission barrier in } ^{266}\text{Hs} \)
- surface energy \( a_{\text{surf}} \) (computed from semi-infinite matter)
- low-lying dipole strength (from photo-absorption cross section \( \sigma_{\text{photoabs}}(E) \))
Optimization of model parameters and subsequent variation
Optimization of model parameters and exploring its variation

model: SHF/RMF
Optimization of model parameters and exploring its variation

parameters: \( \mathbf{p} = (p_1 \ldots p_F) \)

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model: SHF/RMF

observables: $A = \langle \hat{A} \rangle = A(\mathbf{p})$
Optimization of model parameters and exploring its variation

parameters: \( p = (p_1 \ldots p_F) \)
partly NMP:
\( K, \frac{m^*}{m}, J, \kappa, L \)

model: SHF/RMF

observables: \( A = \langle \hat{A} \rangle = A(p) \)
Optimization of model parameters and exploring its variation

\[ \mathbf{p} = (p_1 \ldots p_F) \]

partly NMP:
\[ K, \frac{m^*}{m}, J, \kappa, L \]

model: \text{SHF/RMF}

fit observables:
\[ A_{\text{fit}} = A_{\text{fit}}(\mathbf{p}) \]

predicted observables:
\[ A_{\text{pred}} = A_{\text{pred}}(\mathbf{p}) \]

\[ \chi^2(\mathbf{p}) = \text{least squares error} \]

feedback to minimize \[ \chi^2(\mathbf{p}) \]

error on data:
\[ \Delta A_{\text{exp}} \]

uncertainties:
\[ \Delta \mathbf{p}, \Delta K, \Delta m^*/m, \Delta J, \Delta \kappa, \Delta L \]

particularly large are
\[ \Delta J, \Delta \kappa, \Delta L \quad (= \text{isovector NMP}) \]

\[ \Rightarrow \text{study effect of variation } \Delta J \text{P} . \]
Optimization of model parameters and exploring its variation

parameters: $p = (p_1 \ldots p_F)$
partly NMP: $K, \frac{m^*}{m}, J, \kappa, L$

model: SHF/RMF

exp. fit data: $A_{\text{exp}}$

fit observables: $A_{\text{fit}} = A_{\text{fit}}(p)$

predicted observables: $A_{\text{pred}} = A_{\text{pred}}(p)$
Optimization of model parameters and exploring its variation

\[
\chi^2(p) = \text{least squares error}
\]

Parameters:
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Partly NMP:
\[ K, \frac{m^*}{m}, J, \kappa, L \]

Model: SHF/RMF

Predicted observables:
\[ A_{\text{pred}} = A_{\text{pred}}(p) \]

Fit observables:
\[ A_{\text{fit}} = A_{\text{fit}}(p) \]

Experimental fit data:
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Error on data:
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Uncertainties:
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Uncertainties:
\[ \Delta A_{\text{pred}} \]

Particularly large are \( \Delta J, \Delta \kappa, \Delta L \) (=isovector NMP)

\[ \Rightarrow \text{study effect of variation} \]

\[ P \cdot G \text{ Reinhard (Inst.Theor.Physik, Erlangen)} \]

Isovector observables in nuclear mean-field theories

9. July 2013
Optimization of model parameters and exploring its variation

\[ \chi^2(p) = \text{least squares error} \]

Feedback to minimize

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Parameters:
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Partly NMP:
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Optimization of model parameters and exploring its variation

\[ \chi^2(p) = \text{least squares error} \]

feedback to minimize \( \chi^2(p) \)

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particularly large are
\[ \Delta J, \Delta \kappa, \Delta L (=\text{isovector NMP}) \]

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\[ \chi^2(p) = \text{least squares error} \]

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**exp. fit data:**  
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**error on data:**  
\( \Delta A_{\exp} \)

**uncertainties:**  
\( \Delta A_{\text{pred}} \)

particularly large are  
\( \Delta J, \Delta \kappa, \Delta L \) (=isovector NMP)

\[ \Rightarrow \text{study effect of variation} \Delta J \]
Trend analysis: dedicated variation of $J$

- **Model:** SHF/RMF

- **Parameters:**
  - $\mathbf{p} = (p_1 \ldots p_F)$
  - Partly NMP: $K, \frac{m^*}{m}, J, \kappa, L$

- **Fit Observables:**
  - $A_{\text{fit}} = A_{\text{fit}}(\mathbf{p})$

- **Predicted Observables:**
  - $A_{\text{pred}} = A_{\text{pred}}(\mathbf{p})$

- **Exp. Fit Data:** $A_{\text{exp}}$

- **Chi-Squared:** $\chi^2(\mathbf{p}) = \text{least squares error}$

- **Feedback:**
  - To minimize $\chi^2(\mathbf{p})$

- **Fix $J$ during fit $\implies$ produce parameter set $\mathbf{p}(J)$ for a series of $J$
Trend analysis: dedicated variation of $J$

- **Parameters**: $p = (p_1 \ldots p_F)$
  - partly NMP: $K, \frac{m^*}{m}, J, \kappa, L$

- **Model**: SHF/RMF

- **Fit Observables**: $A_{\text{fit}} = A_{\text{fit}}(p)$

- **Predicted Observables**: $A_{\text{pred}} = A_{\text{pred}}(p; J)$

- **Experimental Fit Data**: $A_{\text{exp}}$

- **Feedback to Minimize**: $\chi^2(p)$

- **Fix $J$ during fit**: produce parameter set $p(J)$ for a series of $J$

- **Trends** $A = A(J)$ reveal dependences
Correlation analysis: propagate $\Delta \mathbf{p}$ to mixed variances $\Delta^2(A_1, A_2)$

**Parameters:**

$\mathbf{p} = (p_1 \ldots p_F)$

Partly NMP:

$K, \frac{m^*}{m}, J, \kappa, L$

**Uncertainties:**

$\Delta \mathbf{p}, \Delta K, \Delta \frac{m^*}{m}, \Delta J, \Delta \kappa, \Delta L$

**Model:**

SHF/RMF

**Explicit fit data:** $A_{\text{exp}}$

**Error on data:** $\Delta A_{\text{exp}}$

**Fit observables:**

$A_{\text{fit}} = A_{\text{fit}}(\mathbf{p})$

**Predicted observables:**

$A_{\text{pred}} = A_{\text{pred}}(\mathbf{p})$

$\Delta^2 A_{\text{pred}} = \sum_{ij} \partial p_i A C_{ij}^{-1} \partial p_j A$

**Feedback to minimize $\chi^2(\mathbf{p})$**

**Key quantity:**

Covariance matrix $C_{ij}^{-1} = \partial p_i \partial p_j \chi^2$

E.g. $\Delta p_i = \sqrt{C_{ii}}$

**Reasonable range of $\mathbf{p}$:**

$\chi^2(\mathbf{p}) = \chi^2_{\text{min}} + 1$
Correlation analysis: propagate $\Delta p$ to mixed variances $\Delta^2(A_1, A_2)$

- **Model**: SHF/RMF
- **Parameters**: $p = (p_1 \ldots p_F)$, partly NMP: $K, \frac{m^*}{m}, J, \kappa, L$

**Fit observables**: $A_{\text{fit}} = A_{\text{fit}}(p)$

**Predicted observables**: $A_{\text{pred}} = A_{\text{pred}}(p)$

**Mixed variance**: $\Delta^2(AB) = \sum_{ij} \partial_{p_i} A C_{ij}^{-1} \partial_{p_j} B$

**Covariance**: $c_{AB} = \frac{\Delta^2(AB)}{\sqrt{\Delta^2(AA) \Delta^2(BB)}}$

- $c_{AB} = 1 \leftrightarrow$ highly correlated
- $c_{AB} = 0 \leftrightarrow$ uncorrelated

**Key quantity**: Covariance matrix $C_{ij}^{-1} = \partial_{p_i} \partial_{p_j} \chi^2$

- $\Delta p_i = \sqrt{C_{ii}}$
The influence of $J = a_{\text{sym}}$ on observables
Trends of nuclear matter properties (NMP) with $a_{\text{sym}}$ for the case of SHF

$K$, $m^*/m$, $\kappa$ independent of $J (= a_{\text{sym}}) \implies$ four independent model parameters

$L =$ slope of $J$ strongly linked with $J \iff$ hidden correlation in data (and model)
Trends of nuclear matter properties (NMP) with $a_{\text{sym}}$ – SHF and RMF

for all cases: $K$, $m^*/m$, $\kappa$ independent of $a_{\text{sym}}$, and $L$ strongly linked with $J$

large differences SHF $\leftrightarrow$ RMF: mass parameters $m^*/m$ and $\kappa$
Correlations of $a_{\text{sym}}$ with NMP, for the case of SHF

Again: $K$, $m^*/m$, $\kappa$ perfectly independent; but strong correlation with $L$, $\partial_e E_{\text{neut}}/N$ surface properties $a_{\text{surf}}$, $a_{\text{surf, sym}}$ show a mixed picture
Good isovector observables (in $^{208}$Pb): weak-charge formfactor $F_W$, neutron skin $r_n - r_p$, polarizability $\alpha_D$, superheavy elements uncorrelated, neutron rich nuclei & fission somewhat correlated.
Correlations of $a_{\text{sym}}$ with NMP and obs. in finite nuclei – SHF & RMF

<table>
<thead>
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<th>Correlation with $a_{\text{sym}}$</th>
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<td>$a_{\text{surf, sym}}$</td>
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RMF models show nearly the same correlations as SHF except for surface properties and fission.

Trends of isovector observables in $^{208}$Pb with $a_{\text{sym}}$

for each model: clear trends with $a_{\text{sym}}$

SHF $\leftrightarrow$ RMF-PC: same slope, different offset

RMF-DD: even different slope $\leftrightarrow$ different $\rho$-dependence

(but small variance of $a_{\text{sym}}$)
Trends of observables in $^{208}\text{Pb}$ against each other

Isovector observables in $^{208}\text{Pb}$ line up nicely with each other. Inter-correlation seems better than correlation with $a_{\text{sym}}$. 

The weak-charge formfactor and correlation with $r_n$
Weak-charge formfactor with uncertainty, for $^{208}\text{Pb}$ with SV-min

$$F_W(q) = G_n^Z(q)F_n(q) + G_p^Z(q)F_p(q)$$

$G_{p/n}^Z = \text{weak-charge FF nucleon}$

$F_{p/n} = \text{FF of } \rho_{p/n}(r)$
Weak-charge formfactor with uncertainty, for $^{208}\text{Pb}$ with SV-min

$$F_W(q) = G_Z^n(q)F_n(q) + G_Z^p(q)F_p(q)$$

$G_Z^{p/n} = \text{weak-charge FF nucleon}$

$F_{p/n} = \text{FF of } \rho_{p/n}(r)$

-dominated by neutron formfactor $F_n$

$\leftrightarrow$ measure neutron radius $r_n$
Weak-charge formfactor with uncertainty, for $^{208}\text{Pb}$ with SV-min

$$F_W(q) = G^Z_n(q)F_n(q) + G^Z_p(q)F_p(q)$$

$G^Z_{p/n}$ = weak-charge FF nucleon

$F_{p/n} = $ FF of $\rho_{p/n}(r)$

dominated by neutron formfactor $F_n$

$\leftrightarrow$ measure neutron radius $r_n$

extrapolation uncertainties depend on $q$

large at PREX point $q_{\text{PREX}} = 0.475$/fm

$\implies$ PREX provides new information
Weak-charge formfactor and its correlation with neutron radius $r_n$

Extrapolation uncertainties depend on $q$ large at PREX point $q_{\text{PREX}} = 0.475/\text{fm}$

$\Rightarrow$ PREX provides new information

$F_W$ highly correlated with $r_n$ at low $q$

Lack of correlation near first zero of $F_W$

SHF and RMF very similar
A few words about low-lying dipole strength ("pygmy region")
“Pygmy strength” = low lying peaks in isovector dipole spectrum

pygmy "resonance" = bunches of dipole strength safely below GDR region

various interpretations:
1) 1ph structures
2) collective motion of neutron surface versus bulk

(SHF surveys favor option 1)

choose a cut off at \( E_{\text{cut}} = 9.5 \text{ MeV} \) to define

\[
\text{"pygmy strength"} = \int_{0}^{E_{\text{cut}}} \sigma_{\text{photoabs}}(E)/E \, dE
\]
Transition formfactor $f_{\text{trans}}(q, E)$ for $L = 1$ modes in $^{208}\text{Pb}$

**$^{208}\text{Pb, SV-bas, L=1 modes}$**

- **$T=0$ transition formfactor /4**
- **$T=1$ transition formfactor *2**

Strong $T = 0$ modes, $q \approx 0.6$

$(q = 0$ occupied by c.m. mode)
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low $E$ dipoles: mixed modes, predominantly isoscalar, nonetheless useful info on NMP

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$q = 0$ occupied by c.m. mode
Trends of low-lying dipole strength with nuclear matter parameters

\[
\int_{E_{\text{cut}}}^E dE \frac{\sigma(E)}{E}
\]

Pygmy strength \(^{208}\text{Pb}\) [arb.u.]

- Effective mass \(m^*/m\)
- TRK sum rule \(\kappa\)
- Incompressibility \(K\) [MeV]
- Symmetry energy [MeV]

Some dependence on each NMP \(\Rightarrow\) mixed info on NMP

Trends of low-lying dipole strength with nuclear matter parameters

\[ \int_0^{E_{\text{cut}}} dE \frac{\sigma(E)}{E} \]

Pygmy str.: some dependence on each NMP

\[ \Rightarrow \]
mixed info on NMP

Correlation of $\int_0^E dE' \sigma(E') E'^n$ with nuclear matter parameters

**correlation: integrated dip. strength with NMP, $^{208}\text{Pb}$, SV-min**

with T=1 dipole, $E^{-1}$ weighted

$E^{-1}$ weight favors $a_{\text{sym}}$ for $E \to \infty$

$E^1$ weight favors $\kappa$ for $E \to \infty$
Correlation of \( \int_0^E dE' \sigma(E') E''^n \) with nuclear matter parameters

**Correlation: integrated dipole strength with NMP, \(^{208}\text{Pb}, \text{SV-min} \)**

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\[ E^+1 \text{ weight favors } \kappa \text{ for } E \to \infty \]

GDR region favors \( \kappa \) for all \( E \) weights

\[ K \quad m^*/m \quad a_{\text{sym}} \quad \kappa_{\text{TRK}} \]
Correlation of $\int_0^E dE' \sigma(E') E''^n$ with nuclear matter parameters

**correlation:** integrated dipole strength with NMP, $^{208}$Pb, SV-min

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$E^{-1}$ weight favors $a_{\text{sym}}$ for $E \to \infty$

$E^{1}$ weight favors $\kappa$ for $E \to \infty$

GDR region favors $\kappa$ for all $E$ weights

Pygmy region yields mixed info about $a_{\text{sym}}$ and $\kappa$ $\Rightarrow$ useful data in combined analysis

(open problem: robust choice of cutoff energy $E_{\text{cut}}$)
Conclusions

Exploring correlations: SHF/RMF are adjusted to data by $\chi^2$ fits. We exploit methods of error propagation in $\chi^2$ technique.

Observables: We concentrate on static isovector observables $\rightarrow$ symmetry energy $\alpha_{sym} \equiv J_T$.

Trends with $J_T$: SHF and RMF-PC have similar trends, but can differ in offset $\leftrightarrow$ relativistic effect? RMF-DD has even different trends $\leftrightarrow$ much different density dependence?

Correlation with $J_T$: Group of highly coorrelated (static) isovector observables: polarizability $\alpha_D$, neutron radius $r_n$, skin $r_n - r_p$, weak-charge formfactor $F_W(q_{PREX})$.

Weak-charge formfactor $F_W$: $\chi^2$ uncertainties depend on $q$, maximal at $q_{PREX} \leftrightarrow$ PREX data most informative.

Low $E_d$ dipole strenth: Dominated by isoscalar $L=1$ modes, spread over several distinct modes. Integrated dipole strength yields combined information on $\alpha_{sym}$ and $\kappa$. 
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