Freeze-out parameters: lattice meets experiment

Claudia Ratti

Università degli Studi di Torino and

INFN, Sezione di Torino

In collaboration with R. Bellwied, S. Borsanyi, Z. Fodor, S. Katz, S. Krieg, K. Szabo
Motivation

✦ We live in a very exciting era to understand the fundamental constituents of matter and the evolution of the Universe

✦ We can create the deconfined phase of QCD in the laboratory

✦ Lattice QCD simulations: unprecedented levels of accuracy
  - physical quark masses
  - several lattice spacings → continuum limit

✦ The joint information between theory and experiment can help us to shed light on the QCD phase diagram
Fluctuations of conserved charges

- The deconfined phase of QCD can be reached in the laboratory

- Need for unambiguous observables to identify the phase transition

  - fluctuations of conserved charges (baryon number, electric charge, strangeness)

- A rapid change of these observables in the vicinity of $T_c$ provides an unambiguous signal for deconfinement

- They can be calculated on the lattice as combinations of quark number susceptibilities

- They can be directly compared to experimental measurements
The observables under study

Fluctuations (susceptibilities) are defined as follows:

\[ \chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p / T^4}{\partial (\mu_B / T)^l \partial (\mu_S / T)^m \partial (\mu_Q / T)^n}. \]

The chemical potentials are related:

\[ \mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q; \]
\[ \mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q; \]
\[ \mu_s = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q - \mu_S. \]

Quadratic fluctuations:

\[ \chi^{X} = \frac{1}{VT^3} \langle N_X^2 \rangle \]

Correlators between different charges:

\[ \chi^{X Y}_{11} = \frac{1}{VT^3} \langle N_X N_Y \rangle. \]
Physical meaning

✦ Diagonal susceptibilities measure the response of quark densities to an infinitesimal change in the chemical potential

$$\chi^X_2 = \frac{\partial^2 p/T^4}{\partial (\mu_X/T)^2} = \frac{\partial}{\partial (\mu_X/T)} \left( \frac{n_X}{T^3} \right)$$

=> A rapid increase of these observables in a certain temperature range signals a phase transition

✦ Non-diagonal susceptibilities measure the correlation between different quark flavors

$$\chi^{XY}_{11} = \frac{\partial^2 p/T^4}{\partial (\mu_X/T)\partial (\mu_Y/T)} = \frac{\partial}{\partial (\mu_Y/T)} \left( \frac{n_X}{T^3} \right)$$

=> They can provide information about bound-state survival above the phase transition
Results: susceptibilities of baryon number and electric charge

\[ \chi_2^B = \frac{1}{9} \left( 2\chi_2^u + \chi_2^s + 2\chi_{11}^{ud} + 4\chi_{11}^{us} \right); \quad \chi_2^Q = \frac{1}{9} \left( 5\chi_2^u + \chi_2^s - 4\chi_{11}^{ud} - 2\chi_{11}^{us} \right) \]

- rapid rise around \( T_c \)
- They reach \( \sim 90\% \) of ideal gas value at large temperatures

WB collaboration, JHEP (2012)
Strange quark susceptibilities have their rapid rise at larger temperatures compared to the light quark ones.

They rise more slowly as a function of $T$.

There is a difference of $\sim 15 - 20$ MeV between the inflection points of the two curves.

WB collaboration, JHEP (2012)
Are there bound states in the QGP?

- Comparison of lattice to PNJL \( (C.R., R. Bellwied, M. Cristoforetti, M. Barbaro, PRD (2012)) \)
  - PNJL MF: pure mean field calculation
  - PNJL PL: mean field plus Polyakov loop fluctuations
  - PNJL MC: full Monte Carlo result with all fluctuations taken into account
  - The red curve falls on the blue for \( V \to \infty \)

- Even the inclusion of fluctuations is not enough to describe lattice data above \( T_c \)

- There is space for a contribution from bound states
Freeze-out temperature from experiment

- Fit to yields of identified particles: Statistical Hadronization Model (SHM)
- Model-dependent. Parameters: freeze-out temperature and chemical potential
Caveats

✦ Lattice results for susceptibilities are first-principle calculations

⇒ However, $T_c$ cannot be univocally defined

✦ The experimental results are fitted by means of the Statistical Hadronization Model

✦ It would be nice to have a direct comparison between first-principle calculations and experimental results
Higher order susceptibilities and ratios

✦ susceptibilities are defined as follows:

\[
\chi_{BSQ}^{lmn} = \frac{\partial^{l+m+n} p/T^4}{\partial (\mu_B/T)^l \partial (\mu_S/T)^m \partial (\mu_Q/T)^n}.
\]

✦ we are now interested in higher order susceptibilities and in ratios

⇒ Ratios have a very peculiar shape which allows to unambiguously spot the transition

⇒ They can be directly related to an experimental measurement: no need for model interpretation!
Relating lattice results to experimental measurement

The first four cumulants are:

\[ \chi_1 = \langle (\delta x) \rangle \]
\[ \chi_2 = \langle (\delta x)^2 \rangle \]
\[ \chi_3 = \langle (\delta x)^3 \rangle \]
\[ \chi_4 = \langle (\delta x)^4 \rangle - 3\langle (\delta x)^4 \rangle^2 \]

We can relate them to higher moments of multiplicity distributions:

Mean: \[ M = \chi_1 \]

Variance: \[ \sigma^2 = \chi_2 \]

Skewness: \[ S = \chi_3 / \chi_2^{3/2} \]

Kurtosis: \[ \kappa = \chi_4 / \chi_2^2 \]

\[ S\sigma = \chi_3 / \chi_2 \]

\[ \kappa\sigma^2 = \chi_4 / \chi_2 \]

\[ M / \sigma^2 = \chi_1 / \chi_2 \]

\[ S\sigma^3 / M = \chi_3 / \chi_1 \]

F. Karsch (2012)
Thermometer and Baryometer

✦ $R_{31}^Q$: thermometer

\[
R_{31}^Q(T, \mu_B) = \frac{\chi_3^Q(T, \mu_B)}{\chi_1^Q(T, \mu_B)} = \frac{\chi_3^B(T, 0) + \chi_4^Q(T, 0)q_1(T) + \chi_3^S(T, 0)s_1(T)}{\chi_{11}^B(T, 0) + \chi_2^Q(T, 0)q_1(T) + \chi_{11}^S(T, 0)s_1(T)} + \mathcal{O}(\mu_B^2)
\]

✦ Expand numerator and denominator around $\mu_B = 0$: ratio is independent of $\mu_B$

✦ $R_{12}^Q$: baryometer

\[
R_{12}^Q(T, \mu_B) = \frac{\chi_1^Q(T, \mu_B)}{\chi_2^Q(T, \mu_B)} = \frac{\chi_{11}^B(T, 0) + \chi_2^Q(T, 0)q_1(T) + \chi_{11}^S(T, 0)s_1(T)}{\chi_2^Q(T, 0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).
\]

✦ Expand numerator and denominator around $\mu_B = 0$: ratio is proportional to $\mu_B$
Freeze-out parameters

Experimental measurement I

Star Collaboration: arXiv 1212.3892

Claudia Ratti
$S^Q \sigma^Q = \frac{\chi^Q_3}{\chi^Q_2}$. 

Average of two most central measurements over $\sqrt{s} = 27, 39, 62.4$. 

Star Collaboration: arXiv 1212.3892
Freeze-out parameters

Extracting freeze-out parameters


✧ Upper limit: $T_f \leq 157 \pm 4$ MeV

<table>
<thead>
<tr>
<th>$\sqrt{s}[GeV]$</th>
<th>$\mu_B^f [MeV]$</th>
</tr>
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<tbody>
<tr>
<td>62.4</td>
<td>44(5)</td>
</tr>
<tr>
<td>39</td>
<td>75(7)</td>
</tr>
<tr>
<td>27</td>
<td>95(9)</td>
</tr>
</tbody>
</table>
Alternative: use baryon number


- Independently extract $T_f$ and $\mu_{B_f}$
- Check thermodynamic consistency
- Experimental measurement still missing
Freeze-out parameters

Strange vs light thermometer

R. Bellwied et al.: PRL (2013)

 Flavor-specific kurtosis confirms separation between light and strange quarks

\[ w_f = \chi_{13}^{B_f} - \chi_{11}^{B_f} \] more sensitive to flavor content: in the hadronic phase it only receives contribution from hadrons with more than one quark of flavor \( f \)
Freeze-out parameters

Charm quark number susceptibilities

\[ \chi^c_2 = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_c \partial \mu_c} \bigg|_{\mu_i=0} \]

- Charm susceptibilities rise at much larger temperatures compared to the light quark ones.
- Their rise with temperature is much slower.
Conclusions

✧ High precision (continuum limit) lattice QCD results allow to extract the freeze-out parameters of heavy-ion collisions

✧ there are indications of a flavor separation in the transition temperatures

✧ this could lead to a short mixed phase of degrees of freedom in which strange particle formation is dominant

✧ this should lead to measurable effects in the strange hadron yields (evidence from ALICE)

✧ new model-independent comparison between theory and experiment: $\chi_4/\chi_2$

✧ work in progress: define a meaningful experimental measurement

✧ ultimate test: hadronic spectral functions at finite-temperature
Backup slides
Relations between chemical potentials

- $\mu_B, \mu_S$ and $\mu_Q$ are NOT independent:

$$\langle n_S \rangle = 0 \quad \langle n_Q \rangle = \frac{Z}{A} \langle n_B \rangle \quad \Rightarrow \quad \frac{Z}{A} = 0.4$$

- By expanding $n_B, n_S$ and $n_Q$ up to $\mu_B^3$ we get:

$$\mu_Q(T, \mu_B) = q_1(T)\mu_B + q_3(T)\mu_B^3 + ...$$

$$\mu_S(T, \mu_B) = s_1(T)\mu_B + s_3(T)\mu_B^3 + ...$$
Taylor coefficients: results


✦ $\mu_Q$ turns out to be very small

✦ Agreement between WB and BNL-Bielefeld collaborations
Defining the experimental measurement

✧ Problem: we need to go from the $B, Q, S$ basis to the basis of $u, d, s$ quark flavors

✧ In principle we need to measure all light and strange quark final states

⇒ Experimentally impossible, most resonances are too rare and cannot be reconstructed

✧ Most resonances decay one to one to their ground state

✧ Strange weak decays need to be reconstructed

\[
\kappa_s \sigma_s^2 = \kappa \sigma^2(K, K^0, \Lambda, \Xi, \Omega \text{ incl. } K^*, \Lambda^*, \Sigma, \Xi^*)
\]
\[
\kappa_u \sigma_u^2 = \kappa \sigma^2(\pi, \rho \text{ incl. } \rho, \omega, \Delta, N^*)
\]

Ongoing project with P. Alba, W. M. Alberico, R. Bellwied, M. Bluhm, D. Chinellato and M. Weber
Freeze-out parameters

Charm quark number susceptibilities

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- charm susceptibilities rise at much larger temperatures compared to the light quark ones
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Possible interpretations

✦ survival of open charm hadrons up to $T \approx 2T_c$?

✦ HRG results agree with the lattice up to the inflection point in the data
Possible interpretations

- survival of open charm hadrons up to $T \simeq 2T_c$?
- HRG results agree with the lattice up to the inflection point in the data
- thermal excitation of charm quarks takes place at larger temperatures
- ideal gas of charm quarks agrees with lattice

need for non-diagonal quark number susceptibilities
Simple experimental verification

- Yields of strange particles should be enhanced relative to yields of non-strange particles.
- Yields of strange particles should result in a higher temperature than yields of non-strange particles when fitted with a statistical hadronization model (SHM).

R. Preghenella
for ALICE
SQM 2012
arXiv:1111.7080
SHM yield fit

* Simultaneous fit gives $T = 152$ MeV

However, protons are overestimated and strange baryons are underestimated

L. Milano for ALICE, QM 2012
SHM yield fit

- Fitting them separately, a hierarchy is evident

R. Bellwied (2012)
Caveats

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✦ It would be nice to have a direct comparison between first-principle calculations and experimental results
Higher order susceptibilities and ratios

- susceptibilities are defined as follows:

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- we are now interested in fourth order susceptibilities \((\chi_4)\) and in particular in ratios \(\chi_4/\chi_2\)

⇒ Ratios have a very peculiar shape which allows to unambiguously spot the transition

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✦ the first four cumulants are:

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\end{align*}
\]

✦ we can relate them to higher moments of multiplicity distributions:

- variance: \( \sigma^2 = \chi_2 \)
- standard deviation: \( \sigma = \sqrt{\chi_2} \)
- skewness: \( S = \chi_3 / \chi_2^{3/2} \)
- kurtosis: \( \kappa = \chi_4 / \chi_2^2 \)

\[
S\sigma = \chi_3 / \chi_2 \\
\kappa\sigma^2 = \chi_4 / \chi_2
\]

✦ and therefore:

\[
\kappa_B\sigma_B^2 \equiv \frac{X_4^{B,\mu}}{X_2^{\mu}} = \frac{X_4^{B}}{X_2^{B}(T)} \left[ \frac{1 + \frac{1}{2} \frac{X_6^{B}(T)}{X_4^{B}(T)} (\mu_B/T)^2 + ...}{1 + \frac{1}{2} \frac{X_4^{B}(T)}{X_2^{B}(T)} (\mu_B/T)^2 + ...} \right]
\]

Claudia Ratti (2012)
Preliminary lattice results

- light and strange quark $\chi_4/\chi_2$ still exhibit 20 MeV difference in the transition temperature
- transition is easy to spot: non-monotonic behavior

WB collaboration, preliminary
Defining the experimental measurement

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⇒ Experimentally impossible, most resonances are too rare and cannot be reconstructed

✦ Most resonances decay one to one to their ground state

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Ongoing project with W. M. Alberico, R. Bellwied, M. Bluhm, D. Chinellato and M. Weber
Alternate explanation: non-equilibrium proton annihilation after hadronization

Idea based on enhanced in-medium annihilation cross sections in hadronic transport codes, e.g. UrQMD

Steinheimer, Aichelin, Bleicher, arXiv:1203.5302
Karpenko, Sinyukov, Werner, arXiv:1204.5351
Becattini et al., arXiv:1201.6349

A hydro-based model which modifies the chemical freeze-out temperatures in a species dependent way due to viscous corrections exists as well (but slightly over predicts proton yield and under predicts multi strange yield).


Method to exclude in-medium proton suppression (annihilation models): Compare Npart scaled proton yield as a function of centrality in PbPb and in pp. If proton is suppressed in medium then scaled yield has to drop with centrality.
At RHIC the proton is not suppressed but rather slightly enhanced.

H. Caines for STAR
SQM 2006
nucl-ex/0608008
Comparison with previous lattice data

\[ \chi^u_2 = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_u \partial \mu_u} \bigg|_{\mu_i=0} \]

✧ physical quark masses \( m_s/m_{u,d} \approx 28 \), before it was \( m_s/m_{u,d} \approx 10 \)

✧ finer lattice spacings and continuum limit

✧ the phase transition turns out to be much smoother
Strangeness enhancement in ALICE
HRG calculations are very sensitive to particle composition
There are evidences for deviations from statistical model predictions at the LHC - J/ψ production -

Data: ALICE/PHENIX (forward rapidity) - QM 2011

Data: ALICE / ATLAS / CMS (mid rapidity) - QM 2011

Prediction: Braun-Munzinger, Stachel arXiv:0901.2500

Conclusion:
All datasets (forward and mid-rapidity, low and high pT) show significant J/ψ suppression in central collisions in contradiction to statistical model predictions: possibly no common freeze-out surface or no strong partonic recombination?
All path approach

✦ Our goal:
  ➡️ determine the equation of state for several pion masses
  ➡️ reduce the uncertainty related to the choice of $\beta^0$

✦ conventional path: A, though B, C or any other paths are possible
✦ generalize: take all paths into account
Finite volume and discretization effects

- finite $V : \frac{N_s}{N_t} = 3$ and 6 (8 times larger volume): no sizable difference
- finite $a$: improvement program of lattice QCD (action observables)
  - tree-level improvement for $p$ (thermodynamic relations fix the others)
  - trace anomaly for three $T$-s: high $T$, transition $T$, low $T$
  - continuum limit $N_t = 6, 8, 10, 12$: same with or without improvement
- improvement strongly reduces cutoff effects: slope $\simeq 0$ (1 $- 2\sigma$ level)
Pseudo-scalar mesons in staggered formulation

✦ Staggered formulation: four degenerate quark flavors (‘tastes’) in the continuum limit

✦ Rooting procedure: replace fermion determinant in the partition function by its fourth root

✦ At finite lattice spacing the four tastes are not degenerate

➤ each pion is split into 16

➤ the sixteen pseudo-scalar mesons have unequal masses

➤ only one of them has vanishing mass in the chiral limit

✦ Scaling starts for \( N_t \geq 8 \).