Aldo Antognini
ETH Zurich
for the
CREMA collaboration
Muonic Atoms

Aldo Antognini
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for the CREMA collaboration

- Muonic hydrogen ($\mu p$)
- Muonic deuterium ($\mu D$)
- Muonic helium ($\mu He^+$)
Measure $\Delta E(2S - 2P)$ → $r_p$ with $\delta r_p = 4 \times 10^{-19}$ m

- Muonic hydrogen ($\mu p$)
- Muonic deuterium ($\mu D$)
- Muonic helium ($\mu \text{He}^+$)
The proton radii puzzle

3 ways to the proton radius

- e-p scattering
- \( ^1H \) precision laser spectroscopy
- \( \mu p \) laser spectroscopy

Pohl et al., Nature 466, 213 (2010)
Proton radius from muonic hydrogen

• Measure $\Delta E_{2P-2S}^{\text{exp}}$ in $\mu_p$ with $u_r = 10^{-5} \leftrightarrow 0.5 \text{ GHz} = \Gamma/20$

• Compute theoretical prediction

$$\Delta E_{2P-2S}^{\text{th}} = 206.0336(15) - 5.2275(10) r_p^2 + 0.0332(20) \text{ [meV]}$$

Comparing theory with experiment $\implies r_p$

The Lamb shift contributions

Fin. size: 3.8 meV

23 meV

F=0

F=1

8.4 meV

F=2

F=1

F=0

2P_3/2

2P_1/2

2S_1/2

206 meV

50 THz

6 \mu m
Principle of the $\mu p$ Lamb shift experiment

- Produce many $\mu^-$

- Stop $\mu^-$ in 1 mbar $H_2$ gas
  $\rightarrow$ $\mu p$ formation
  (1% in the 2S-state with 1 $\mu$s lifetime)

- Fire laser at $\lambda = 6 \mu m$
  $\rightarrow$ to induce $\mu p(2S) \rightarrow \mu p(2P)$ transition

- If laser resonant
  $\rightarrow$ observe 2 keV x-rays

Dedicated low-energy $\mu^-$ beam line

Dedicated laser system with “strange” requirements

2 keV x-ray detectors
The $\mu p$ Lamb shift setup

- **p-beam (1 MW)**
- $\pi$ (MeV)
- $\mu$ (MeV)
- $\mu$ (keV)
- $\mu p(2S)$
- Laser
- X-ray

Diagram:
- H2 target
- 5T solenoid
- Momentum filter
- Pion beam line
- Protons C-target
- Laser
- Ti:Sa cw laser
- Ti:Sa oscillator
- Raman cell
- SHG
- Diode laser
- Disk-laser
- FP cavity

Frequencies:
- 940 nm (2 kW)
- 1030 nm
- 515 nm
- 708 nm
- 1.0 $\mu$m
- 1.6 $\mu$m
- 6.0 $\mu$m

Water line scan:
Laser frequency known with 300 MHz uncertainty

Discrepancy:
5.0σ ↔ ~ 75 GHz ↔ δν/ν = 1.5 × 10⁻³

Pohl et al., Nature 466, 213 (2010)
We have measured two transitions in $\mu p$
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- Considering the two measurements separately

Two independent determinations of $r_p$

$(\nu_t \rightarrow r_p, \nu_s \rightarrow r_p)$

Consistent results !!!
We have measured two transitions in $\mu p$

- Considering the two measurements separately
  
  Two independent determinations of $r_p$
  
  \[(\nu_t \rightarrow r_p, \nu_s \rightarrow r_p)\]

  Consistent results !!!

- Combining the two measurements
  
  Two measurements $\rightarrow$ determine two parameters
  
  $\nu_t, \nu_s \rightarrow \Delta E_L, \Delta E_{\text{HFS}} \rightarrow r_p, r_Z$

  
  \[
  r_Z = \int d^3r_1 d^3r_2 \rho_E(r_1)\rho_M(r_2) |r_1 - r_2|
  \]

  \[
  \frac{3}{4} \nu_t + \frac{1}{4} \nu_s = \Delta E_L(r_p) + 8.8123 \text{ meV}
  \]

  \[
  \nu_s - \nu_t = \Delta E_{\text{HFS}}(r_Z) - 3.2480 \text{ meV}
  \]
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New $r_p$ does NOT depend on 2S-HFS prediction
We have measured two transitions in $\mu p$!

$$\nu_t = \nu(2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}) \text{ at } \lambda = 6.0 \, \mu m$$

$$\nu_s = \nu(2S_{1/2}^{F=0} - 2P_{3/2}^{F=1}) \text{ at } \lambda = 5.5 \, \mu m$$

Both resonances are 0.3 meV discrepant from predictions using $r_p$ from CODATA
Results on $\mu p$: $r_p$

$\nu(2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2}) = 49881.88(76) \text{ GHz}$

$\nu(2S_{1/2}^{F=0} \rightarrow 2P_{3/2}^{F=1}) = 54611.16(1.05) \text{ GHz}$

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\[ \begin{align*}
r_p &= 0.84087 (26)_{\text{exp}} (29)_{\text{th}} = 0.84087 (39) \text{ fm}
\end{align*} \]


Proton charge radius: $r_p = 0.84087 (26)_{\text{exp}} (29)_{\text{th}} = 0.84087 (39) \text{ fm}$
Proton radius puzzle: What may be wrong?

- Bound-state QED?
- Proton structure?
- Measurements?
- Definition p-radius?
- "New physics"?

More than 150 publications
Politically correct discussion

Everybody is right!..?
$r_p$ puzzle (1): Is the $\mu p$ experiment wrong?

**Systematics?**

- Laser frequency calibration: 300 MHz
- Zeeman effect ($B = 5$ Tesla): 30 MHz
- AC-Stark, DC-Stark shift: $< 1$ MHz
- Doppler shift: $< 1$ MHz
- Pressure shift (1 mbar): 1 MHz

Systematics shift $\sim 1/m$

Finite size shift $\sim m^3$

**Spectroscopy of $pp\mu$ molecules or $p\mu e$ ions?**

Do not exist or too short lived (in 2S state)

Karr and Hilico, PRL 109, 103401 (2012)
Pohl et al., PRL 97, 193402 (2006)

**Frequency mistake by 75 GHz?**

- Huge difference for laser spectroscopy accuracies
- Two ways to calibrate the frequency (consistent)

Discrepancy $= 75$ GHz $\approx 4\Gamma$

Two consistent $\mu p$ transition measurements

$\mu p$ experiment is probably not wrong by 100 $\sigma$
$r_p$ puzzle (2): Is the $\mu p$ theory wrong?

Discrepancy $= 0.31$ meV
Theory uncertainty $= 0.0025$ meV
⇒ $120\delta$(theory) deviation?

$$\Delta E^{\text{th}} = 206.0668(25) - 5.2275(10) r_p^2 \quad [\text{meV}]$$

Pachucki, PRA 60, 3593 (1999)
Borie, arXiv: 1103.1772-v6
Karshenboim et al., PRA 85, 032509 (2012)
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⇒ $120\delta$ (theory) deviation?

$\Delta E^{\text{th}} = 206.0668(25) - 5.2275(10) r_p^2$ [meV]

Are one- or two-loop VP wrong?

Proton shape dependence?
Borie, Miller

Off-shell proton!

Pachucki, PRA 60, 3593 (1999)
Borie, arXiv: 1103.1772-v6
Karshenboim et al., PRA 85, 032509 (2012)
**$r_p$ puzzle (2): Is the $\mu p$ theory wrong?**

- Can we find a p-shape to solve the discrepancy?  
  DeRujula

  YES IF the proton would have charge distributions with very large “tails”: 
  \[
  \Delta E_{\text{finite size}} = \sum_n a_n \langle r^n_p \rangle
  \]

**BUT**

- bound-state QED expansion $\rightarrow$ $a_n$ decreases rapidly  
  Friar, Indelicato

- e-p scattering data $\rightarrow$ $\langle r^n_p \rangle$ sufficiently small for $n < 6$  
  Distler, Miller

- $\chi$PT $\rightarrow$ no large tails possible  
  Pineda
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Could the two-photon exchange explain the discrepancy?

Calculate the TPE contribution via doubly-virtual Compton tensor using dispersion relation. The imaginary part are the measured proton spin-averaged structure functions
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  BUT a subtraction term needed \( T_1(0, Q^2) \):
  at low-\( Q^2 \) (NRQED +\( \alpha, \beta \)...) and high-\( Q^2 \) (QCD) known!
  At intermediate-\( Q^2 \)?

Unknown: Could be MUCH larger as previously assumed
Under control: Direct calc. of whole contribution in LO \( \chi \text{PT} \)
Under control: \( \chi \text{PT} \) expansion to bridge low-\( Q^2 \) to high-\( Q^2 \)
Under control: Sum rule + Regge +...photoabsorbtion data

Hill and Paz, PRL 107, 160402 (2011), Miller
Nevado and Pineda, PRC 77, 035202 (2008)
McGovern and Birse, EPJA 48 120 (2012)
Gorchtein et al, PRA 84, 052501 (2013)
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  McGovern and Birse, EPJA 48 120 (2012)

  \( \Delta E_{\text{sub}} = -0.0042(10) \) meV  \( \leftrightarrow \) Discrepancy=0.3 meV

  Pseudochtein \textit{et al}, PRA 84, 052501 (2013)
$r_p$ puzzle (3): Is H-spectroscopy wrong?

Two measurements $\rightarrow$ two unknown: $R_\infty$ and $L_{1S}^{\text{exp}}$

$\updownarrow$

$L_{1S}^{\text{th}}(r_p) = 8171.636(4) + 1.5645 \, r_p^2$ MHz

$E_nS \simeq \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$
**rp puzzle (3): Is H-spectroscopy wrong?**

Two measurements → two unknown: $R_\infty$ and $L^{\exp}_{1S}$

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\begin{align*}
E_{nS} & \simeq \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3} \\
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\]

Proton charge radius (fm)

$H_{\text{avg}} = 0.8779 \pm 0.0094 \text{ fm}$

$\mu_p : 0.84087 \pm 0.00039 \text{ fm}$
$r_p$ puzzle (3): Is H-spectroscopy wrong?

Two measurements $\rightarrow$ two unknown: $R_\infty$ and $L_{1S}^{\text{exp}}$

\[ L_{1S}^{\text{th}}(r_p) = 8171.636(4) + 1.5645 \, r_p^2 \, \text{MHz} \]

Discrepancy $< 3\sigma$ for individual H meas.

$E_{nS} \simeq \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$

Proton charge radius (fm)

H$_\text{avg}$ = 0.8779 + 0.0094 fm

µp : 0.84087 + 0.00039 fm
\( r_p \) puzzle (5): Is e-p scattering wrong?

\[
\left( \frac{d \sigma}{d\Omega} \right)_{\text{Ros.}} = \left( \frac{d \sigma}{d\Omega} \right)_{\text{Mott}} \frac{1}{1 + \tau} \left( \varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2) \right)
\]

\[
\langle r_p^2 \rangle = -6\hbar^2 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}
\]

Extrapolation to \( Q^2 \to 0 \) required

[Vanderhaegen and Walcher, ArXiv:1008.4225.]
$r_p$ puzzle (5): Is e-p scattering wrong?

\[
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Extrapolation to $Q^2 \to 0$ required

```
Spline fit
Rosenbluth Separation
```

Needs a fit
Model dependence?

Hills and Paz, PRD 82, 113005 (2010)
Bernauer et al, PRL 105, 242001 (2010)

\[
G_E(Q^2) = 1 + \frac{Q^2}{6} \langle r_p^2 \rangle + \frac{Q^4}{120} \langle r_p^4 \rangle + \ldots
\]

- Very low $Q^2$ yields slope but sensitivity is small
- Larger $Q^2$ more sensitive but larger higher-order terms

[Vanderhaegen and Walcher, ArXiv:1008.4225.]
Analysis of e-p, e-n scattering data using VMD and dispersion relations gives radii in agreement with $\mu p$ albeit a larger $\chi^2$. The two transition measurements in $\mu p$ at very different wavelengths are consistent.

Extrapolation of scattering data? $R_\infty$ and higher transitions in H?

CODATA-2010

H/D

e-p, Mainz

dispersion 2007

$\mu p$ 2010

dispersion 2012

e-p, JLab

$\mu p$ 2013
Analysis of e-p, e-n scattering data using VMD and dispersion relations gives radii in agreement with \( \mu_p \) albeit a larger \( \chi^2 \).

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Extrapolation of scattering data? \( R_\infty \) and higher transitions in H?

BSM physics?
$r_p$ puzzle: New physics?

Imply breakdown of muon-electron universality
BUT must evade limitations from other data
$(g - 2)_{\mu/e}$, $\mu e$, $H$, $\mu Si$ spectroscopy, $J/\Psi$, $\pi$, $K$, $\eta$ decay widths, n-scattering . . .

If $r_p$ reveals new physics:

$$\alpha_x = O(10^4 G_F) \quad \text{and} \quad m_x \in [1 - 1000] \text{ MeV}$$

[after Pospelov]
Rp puzzle: New physics?

Models exist which escape the many constrains but at “high price”: targeted coupling and fine tuning

Carlson

\[ m_x \sim \text{MeV} \]

\[ \text{coupling} \sim 10^{-4} \]

Batell, McKeen and Pospelov, PRL 107, 011803 (2011)
Rislow and Carlson, PRD 86, 035013 (2012)
Trucker-Smith and Yavin, PRD 83, 101702 (2011)

Window for new physics is very small. BUT more “natural” extensions could come into play IFF

\[ r_p^H < r_p^{\mu p} < r_p^{\text{scatt}} \]

→ e.g. dark photons

Pospelov

• If the proton radius puzzle is caused by muon-electron universality breakdown

\[ \mu \text{He}^+ \] spectroscopy will reveal it!

But also \[ \mu \text{D} \] spectroscopy can provide important insights
The 3 measured $\mu d$ transitions

The fit is done assuming the 2P splitting predicted by theory.

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Deuteron radius from $\mu d$ (preliminary)

- Three transitions frequencies measured in $\mu d$
- 2P fine and hyperfine contributions from theory

Borie, Martynenko

$\Rightarrow$ Fit Lamb shift and 2S-HFS

\[
\begin{align*}
\mu d: & \quad \Delta E_{LS}^{\text{exp}} = 202.8759(34) \text{ meV (prel!)} \\
\mu p: & \quad \Delta E_{LS}^{\text{exp}} = 202.3706(23) \text{ meV}
\end{align*}
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<td>= ...</td>
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- Pachucki TPE term should be completed with:
  - finite-size of the nucleons (0.029 meV)?
  - neutron polarisabilities (0.040 meV)?

- Pachucki, Ji et al., and Friar agree on the 2% level
- Ongoing work of Carlson, Gorchtein and Vanderhaegen using inelastic data and dispersion relations.

- NO $R^3_{(2)}$ term (third Zemach term) in $\mu d$. \text{ Pachucki, PRL 106, 193007 (2011)}

Exact cancellation for point-like nucleons between elastic (third Zemach) and part of the inelastic contributions
Deuteron radius from $\mu d$ (preliminary)

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  Priv. Com. Friar

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Deuteron radius from $\mu d$ and $\mu p$ (preliminary)

H-D iso-shift: $r_d^2 - r_p^2 = 3.82007(65) \text{ fm}^2$

$\mu p: \quad r_p = 0.84087(39) \text{ fm}$

$\Rightarrow r_d = 2.12771(22) \text{ fm}$
Deuteron radius from $\mu_d$ and $\mu_p$ (preliminary)

H-D iso-shift: $r_d^2 - r_p^2 = 3.820 \pm 0.07(65) \text{ fm}^2$

$\mu_p$: $r_p = 0.84087(39) \text{ fm}$

$\Rightarrow r_d = 2.12771(22) \text{ fm}$

Directly from $\mu_d$ spectroscopy using $\mu_d$ polarizability with $\pm 0.03 \text{ meV}$

- double counting (th)?
- missing terms (th)?
- shifts due to close levels (exp)?

CODATA-2010

CODATA D + e-d

e-d scatt.

n-p scatt.

Deuteron charge radius [fm]
Deuteron radius from $\mu d$ and $\mu p$ (preliminary)

H-D iso-shift: $r_d^2 - r_p^2 = 3.82007(65)$ fm$^2$ \[ \Rightarrow r_d = 2.12771(22)$ fm

Directly from $\mu d$ spectroscopy using $\mu d$ polarizability with $\pm 0.03$ meV

- double counting (th)?
- missing terms (th)?
- shifts due to close levels (exp)?

IFF new physics $\rightarrow$ not coupling to neutrons

Muonic $r_d$ and $r_p$ are consistent!
µHe$^+$ Lamb shift

Measure $\Delta E(2S-2P)$ in $\mu^3\text{He}^+$ and $\mu^4\text{He}^+$ with 50 ppm

$\Rightarrow r^3\text{He}$ and $r^4\text{He}$ with $u_r = 3 \times 10^{-4} \iff 0.0005$ fm

if polarisability contribution known with $u_r = 5\%$

Antognini et al., Can. J. Phys. 89, 47 (2011)
$\mu^+ \text{He}^+$ Lamb shift

Measure $\Delta E(2S-2P)$ in $\mu^3\text{He}^+$ and $\mu^4\text{He}^+$ with 50 ppm

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Proton radius puzzle - new muonic force?

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Antognini et al., Can. J. Phys. 89, 47 (2011)
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$\Rightarrow$

$r^3\text{He}$ and $r^4\text{He}$ with $u_r = 3 \times 10^{-4} \iff 0.0005$ fm

if polarisability contribution known with $u_r = 5\%$

Proton radius puzzle
- new muonic force?

Benchmark for few-nucleon theories
- absolute radii of $^3\text{He}$, $^4\text{He}$ and $^6\text{He}$, $^8\text{He}$ via isotopic shifts

Antognini et al., Can. J. Phys. 89, 47 (2011)

R. van Rooij et al. Science 333, 196 (2011)
Cancio Pastor et al., arXiv:1201.1362
Müller, Wang, Shiner...
\[\mu \text{He}^+ \text{ Lamb shift}\]

Measure \(\Delta E(2S-2P)\) in \(\mu^3 \text{He}^+\) and \(\mu^4 \text{He}^+\) with 50 ppm

\[\Rightarrow r^3 \text{He} \text{ and } r^4 \text{He} \text{ with } u_r = 3 \times 10^{-4} \equiv 0.0005 \text{ fm}\]

Proton radius puzzle - new muonic force?

Benchmark for few-nucleon theories - absolute radii of \(^3\text{He}, \text{ } ^4\text{He}\) and \(^6\text{He}, \text{ } ^8\text{He}\) via isotopic shifts

if polarisability contribution known with \(u_r = 5\%\)

Antognini et al., Can. J. Phys. 89, 47 (2011)

R. van Rooij et al. Science 333, 196 (2011)
Cancio Pastor et al., arXiv:1201.1362
Müller, Wang, Shiner...

Enhanced bound-state QED test when combined with \(\text{He}^+(1S-2S)\)
- Finite size \(\sim Z^4 R^2\)
- Bohr structure \(\sim Z^2 R_\infty\)
- Challenging QED contributions \(\sim (Z\alpha)^5...6\)

[MPQ and Amsterdam]
Why testing bound-state QED?

- **Free QED**
  \[ a_e = C_1 \left( \frac{\alpha}{\pi} \right) + C_2 \left( \frac{\alpha}{\pi} \right)^2 + C_3 \left( \frac{\alpha}{\pi} \right)^3 + C_4 \left( \frac{\alpha}{\pi} \right)^4 + C_5 \left( \frac{\alpha}{\pi} \right)^5 + \Delta(\text{had.,...}) \]

- **Bound-state QED**
  - Binding effects \((Z\alpha)\) bad convergence, all-order approach/expansion
  - Radiative corrections \((\alpha\) and \(Z\alpha)\)
  - Recoil corrections \((m/M\) and \(Z\alpha)\) relativity \(\Leftrightarrow\) two-body system
  - Radiative–recoil corrections \((\alpha, m/M\) and \(Z\alpha)\)
  - Nuclear structure corrections

  \[ \rightarrow \text{Cannot develop the calculation in a systematic way} \]
  \[ \rightarrow \text{Corrections are mixed up: } \alpha^x \cdot (Z\alpha)^y \cdot (m/M)^z \]
  \[ \rightarrow \text{Difficulty in finding out the desired order of corrections} \]

**New development: NRQED**

<table>
<thead>
<tr>
<th>QED</th>
<th>g − 2 free particle particle mass only perturbative around free particle</th>
<th>Lamb shift bound-state particle three scales, hierarchy non-perturbative</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD</td>
<td>deep inelastic scattering pQCD</td>
<td>hadron lattice, Chiral perturbation</td>
</tr>
</tbody>
</table>

[after Nio]
### Precision test of $B_{50}$, $B_{60}$... contributions

<table>
<thead>
<tr>
<th>Term</th>
<th>H [kHz]</th>
<th>He$^+$ [kHz]</th>
<th>ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta E_{2S-1S}$</td>
<td>$2.466 \times 10^{12}$</td>
<td>$9.869 \times 10^{12}$</td>
<td>$Z^2 = \frac{3}{4} Z^2 R_{\infty} + \delta(L_{1S} - L_{2S})$</td>
</tr>
<tr>
<td>$\delta(L_{1S} - L_{2S})^{\text{exp}}$ (from $\delta R_{\infty}$)</td>
<td>16 (2.2 ppm)</td>
<td>65 (0.7 ppm)</td>
<td>$Z^2 = \delta(\Delta E_{2S-1S} - \frac{3}{4} Z^2 R_{\infty})$</td>
</tr>
<tr>
<td>$(L_{1S} - L_{2S})^{\text{th}}$</td>
<td>7127887(44)</td>
<td>93856127(348)</td>
<td>$Z^{3.7}$ [Jentschura, 2006]</td>
</tr>
<tr>
<td>$\delta(L_{1S} - L_{2S})^{\text{th}}$</td>
<td>(6.3 ppm)</td>
<td>(3.7 ppm)</td>
<td></td>
</tr>
<tr>
<td>$B_{60}$ and $B_{7i}$ terms</td>
<td>$-8(3)$</td>
<td>$-543(185)$</td>
<td>$Z^6...$</td>
</tr>
<tr>
<td>nuclear size ($p$, $^4$He)</td>
<td>1102(44)</td>
<td>62079(295)</td>
<td>$Z^4 r^2$</td>
</tr>
</tbody>
</table>

### Notes
- After $\mu p$ and $\mu He$ experiments
- Uncert. of nucl. size: $2(2)$, $40(16)$ $\mu He^+$-pol. 5%, 2%
- Check $B_{60}$ and $B_{7i}$ with $25%$, $7%$, $3%$ $\mu He^+$-pol. 5%, 2%
Precision test of $B_{50}, B_{60}$... contributions

<table>
<thead>
<tr>
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<th>ratio</th>
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</tr>
<tr>
<td>$\delta(L_{1S} - L_{2S})^{\exp}$ (from $\delta R_\infty$)</td>
<td>$16$ ($2.2$ ppm)</td>
<td>$10$ ($0.7$ ppm)</td>
<td>$Z^2$ [$= \delta(\Delta E_{2S-1S} - \frac{3}{4} Z^2 R_\infty)$]</td>
</tr>
<tr>
<td>$(L_{1S} - L_{2S})^{th}$</td>
<td>$7 127 887(44)$</td>
<td>$93 856 127(348)$</td>
<td>$(H_{1S-2S} + \mu p)$</td>
</tr>
<tr>
<td>$\delta(L_{1S} - L_{2S})^{th}$</td>
<td>$7 127 887(44)$</td>
<td>$93 856 127(348)$</td>
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</tr>
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<td>$62 079(295)$</td>
<td>$Z^4 r^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>He$^+$-pol. 5%</th>
<th>He$^+$-pol. 2%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu p$ after experiments</td>
<td>$\mu He^+$</td>
<td>$\mu He^+$</td>
</tr>
<tr>
<td>uncert. of nucl. size</td>
<td>$(2)$</td>
<td>$(16)$</td>
</tr>
<tr>
<td>check $B_{60}$ and $B_{7i}$ with</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu He^+$-pol. 5%</td>
<td>$\mu He^+$-pol. 2%</td>
<td></td>
</tr>
</tbody>
</table>
Few-nucleon theories and He-radius

(a) Few-Nucleon Interactions in $\chi$EFT

<table>
<thead>
<tr>
<th>$2N$ ints</th>
<th>LO</th>
<th>NLO</th>
<th>$N^2$LO</th>
<th>$N^3$LO</th>
</tr>
</thead>
<tbody>
<tr>
<td>typ. momentum breakdown scale</td>
<td>$\ll 1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-Range: correct symmetries and IR degrees of freedom: Chiral Dynamics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short-Range: symmetries constrain contact-ints to simplify UV: Minimal parameter-set</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hierarchy: 2NF-effects $\gg$ 3NF-effects $\gg$ 4NF-effects</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From $r_{He} \rightarrow c_D$ or $c_E$.

Radii are “clean” benchmarks to test few-nucleons theories or to fix LEC.

[Navratil et al., PRL99, 042501 (2007)]
What do we need from you for $\mu \text{He}^+$?

- **2010**

$$
\Delta E_{\text{LS}}^{\text{th}} = 1670.370(10)(600) - 105.322 r_{\text{He}^2}^2 + 1.529 r_{\text{He}^3}^3 \text{ meV}
= 1380.020(10)^{\text{QED}}(600)^{\text{pol}}(1450)^{\text{fin. size}} \text{ meV}

$$

$$
\Delta E_{\text{LS}}^{\exp} = 13xy.lpq(70)^{\text{stat.}} \text{ meV}

\text{Borie, Martynenko}

- **2013**

<table>
<thead>
<tr>
<th>Pol. contribution</th>
<th>$-3.1 \text{meV} \pm 20%$ Friar (1977)</th>
<th>from measured photo-abs cross sections</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-2.47 \text{meV} \pm 6%$ Ji et al. (2013)</td>
<td>from state-of-the-art potentials, ab-initio</td>
</tr>
<tr>
<td>Third-Zemach</td>
<td>cancellation Friar, Ji et al. exact for point-like nucleons</td>
<td></td>
</tr>
<tr>
<td>Missing polarizability</td>
<td>contribution of nucleons</td>
<td></td>
</tr>
</tbody>
</table>

**Ongoing work on polarizability**: Schlesser

- Also missing:
  - Polarizability contr. to Lamb shift for $\mu^3\text{He}^+$
  - Polarizability contr. to HFS for $\mu^3\text{He}^+$
  - Polarizability contr. to HFS for $\mu d$
  - What is the charge distribution?
  - Shape dependence of our results?

<table>
<thead>
<tr>
<th>Aimed experimental accuracy</th>
<th>15 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural linewidth</td>
<td>320 GHz</td>
</tr>
<tr>
<td>Third Zemach</td>
<td>2000 GHz</td>
</tr>
<tr>
<td>Nuclear pol: 2.47meV</td>
<td>600 GHz</td>
</tr>
<tr>
<td>Uncertainty of nucl. pol. (6%)</td>
<td>36 GHz</td>
</tr>
</tbody>
</table>
Data taking for $\mu\text{He}^+$ starting soon

- same beam line (replace H with He)
- disk-laser performance improved
- laser system ready (no Raman cell)
- gas system for $^3\text{He}$ in preparation

We will measure transitions in $\mu\text{He}^+$ and $\mu\text{He}^+$ with $\lambda \in [800, 1000]$ nm

Beam time starts in October 2013
Motivation, summary, outlook

Test of H energy levels
Bound-state QED

Mu = \( \mu^+ e^- \)
Ps = \( e^+ e^- \)

New physics?

Scattering
\( e + p \rightarrow e + p \)
\( e + d \rightarrow e + d \)
\( \mu + p \rightarrow \mu + p \)
\( \gamma + p \rightarrow \gamma + p \)
...

H-spectroscopy

Proton charge radius
Proton Zemach radius
Deuteron charge radius

\( R_\infty = 3.2898419602495(10)(25)10^{15} \) Hz/c
combining \( \mu p \) with H spectroscopy

A. Antognini
ECT*, Trento
01.08.2013 – p. 29
Collaboration

- E08-007 @ JLAB, e-p at very low $Q^2$
- A1-1/12 @ Mainz, e-d at very low $Q^2$
- MUSE @ PSI, $\mu$-p/e-p
- E05-015 and CLASS @ JLAB, test $2\gamma$
- OLYMPUS@ DESY and VEPP3, test $2\gamma$
- Structure functions
- Compton scattering

Rydberg constant

- Flowers @ NPL: $2S - nS, D : n > 4$
- Tan @ NIST: $\text{Ne}^{9+}$
- Hänsch @ MPQ: $2S - 4P$
- Nez @ LKB: $1S - 3S$
- Hessels @ York: $2S - 2P$
- Pachucki: He
- Udem @ MPQ: He$^+$
- Eikema @ Amsterdam: He$^+$

Exotic atoms spectroscopy

- CREMA, $\mu$ He$^+$
- ETHZ-PSI, Muonium and positronium

Theory and theoretical theory

- Bound-state QED
- Few-nucleon theories
- New physics, including weird QCD and QED
- Hadronic effects and proton structure (EFT, $\chi$PT, lattice?...)
- Analysis of scattering data