REACTION MECHANISM FOR REACTIONS INVOLVING ONE OR TWO ACTIVE PARTICLES: A SIMPLE ONE-DIMENSIONAL MODEL

Andrea Vitturi

Dipartimento di Fisica e Astronomia, Padova
• Motivation
• One-particle dynamics
  1. Inelastic scattering and break-up
  2. Transfer process
• Two-particle dynamics
  Competition of one- and two- particle reactions
Basic problem: description of both structure and dynamics of weakly-bound systems with one or more valence particles.

Even considering inert cores, the problem is relatively easy only with one valence particle (one-particle halo), but starts to be more complex with two particles (two-particle halo), becoming extremely complicated with more particles.

For these reasons one typically uses the expedient of resorting to a number of reaction models and approximation schemes (coupled channels, first-order approximation, space truncation, “effective” optical potentials and formfactors, continuum discretization, etc) that need to be tested (not only against experimental data ……).
In addition, one would like to clarify the so-called “reaction mechanism”, namely to describe the process in terms of single or repeated action of the external field in a perturbative expansion.

Typical example: two-particle transfer process. The pair is transferred in a single shot or in a “correlated” sequence of two single-particle transfer through a number of intermediate states?
To simplify the problem, we will assume particles moving just in a one dimension. In spite of the drastic assumption, the problem may maintain the main features and properties of the full three-dimensional case, for example for the description of transfer, inelastic and break-up processes, in particular for the treatment of pairing correlations and their connection with two-particle transfer or two-particle break-up processes.
First case: processes involving just one particle

Our particle is initially sitting on a single-particle level of a one-body potential and feels the action of a second moving potential.

At the end of the process the wave function of the particle is

a) partly inside the initial well (elastic and inelastic processes)

b) partly inside the moving well (transfer process)

c) partly moving outside of the two wells (break-up process)

The different components can be obtained by projecting the final wave function on the eigenfunctions of the two wells, as well as on the continuum states.
\[ i\hbar \frac{d}{dt} \Psi(x, t) = \mathcal{H}(x, t) \Psi(x, t) \]

\[ \mathcal{H}(x, t) = -\frac{\hbar^2}{2\mu} \frac{d^2}{dx^2} + V_T(x) + V_P(x_P(t)) \]

a. Potentials

Let's see it in action:

b. Trajectory

\[ x_P(t) = x + v_{in} t + \frac{1}{2} a t^2 \]
The choice of the parameters entering in the calculation will lead to different structural and kinematical conditions (corresponding to rather different physical situations and simulating different bombarding energy regimes):

a) parameters of the two wells (consequent energies of single-particle states in both potentials)
b) initial condition (selecting one of the single particle state in target potential)
   c) distance of closest approach
d) acceleration at the distance of closest approach
small break-up (13 %)  
final wave function  
no transfer  
n=2 (inelastic) 22 %  
n=1 (elastic) 65 %
Given the simplicity of the problem, we can easily compare the results with those obtained in the "standard" coupled-channel formalism where the Schrödinger equation is solved by expanding the total wave function into a stationary basis

$$\Psi(x, t) = \sum a_n(t) \Phi_n(x)$$

and solving the coupled system of equations for the amplitudes $a_N(t)$

$$i\hbar \frac{da_j}{dt} = \sum_n <j|V_P(t)|n> e^{i\Delta E t} a_n(t)$$

with the proper initial conditions

$$a_1(t = -\infty) = 1, a_{n \neq 1}(t = -\infty) = 0$$

In first-order approximation

$$a_j = -\frac{i}{\hbar} \int <j|V_P(t)|s> e^{i\Delta E t} dt$$
The basis for the coupled-channel description naturally includes all bound states. But in cases involving the excitation to weakly-bound or to unbound states the inclusion of continuum states are essential in the proper description of the evolution of the system. This is normally done by resorting to the procedure of energy discretization.

This can be done in different ways (HO, THO, box, averaging true continuum states in bunches, etc), and each procedure has limits of validity and convergence, to be tested.
Example: Slicing the continuum (in steps of $\Delta E$) a la CDCC
Results of the coupled-channel calculation

<table>
<thead>
<tr>
<th>Final population</th>
<th>Exact</th>
<th>First-order</th>
<th>Coupled-channels (only bound states)</th>
<th>Coupled-channels (including Continuum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=1 elastic</td>
<td>65 %</td>
<td></td>
<td>73 %</td>
<td>65 %</td>
</tr>
<tr>
<td>n-2 inelastic</td>
<td>22 %</td>
<td>29 %</td>
<td>27 %</td>
<td>22 %</td>
</tr>
<tr>
<td>break-up</td>
<td>13 %</td>
<td></td>
<td></td>
<td>13 %</td>
</tr>
</tbody>
</table>
We consider now the case in which the kinematical conditions allow the transfer of part of the wave function to the moving potential.

So, at the end of the process the wave function of the particle is

a) partly inside the initial well (elastic and inelastic processes)
b) partly inside the moving well (transfer process)
c) partly moving outside of the two wells (break-up process)

OBS Since the basis constructed over the target and the projectile are not orthogonal one needs to introduce, in the “equivalent” coupled-channel approach, the so-called dual-basis, which depends on the distance between the two partners and therefore depends on time.
moving well
initial wave function

\[ p = x_0 + at^2 \]
Time evolution

Wave function

Potential
Time evolution

Wave function

Potential

$|\psi|^2$

Energy (MeV)

$x$ (fm)

$T = 50/210$
Time evolution

Wave function

Potential

Energy (MeV)

$|\Psi|^2$

$T = 100/210$
Time evolution

Wave function

Potential

part of the wf transferred to the moving well

\( T = 150/210 \)
Time evolution

Wave function

Potential

\[ |\Psi|^2 \]

\[ T = \frac{180}{210} \]

Energy (MeV)

\[ x \text{ (fm)} \]
Time evolution (final)

Wave function

|\psi|^2

Potential

Energy (MeV)

-30

-20

-10

0

0

20

40

x (fm)

T = 210/210

negligible break-up (~10%)

elastic (~20%)

transfer (~70%)

One node

N=2

transfer

N=2

N=1
Q-value dependence

ΔE

Q-value dependence

ΔE

OBS: Width of Q-value window depends on the velocity of the moving well (acceleration)
Another case: the particle is initially in a weakly-bound orbital

[Diagram showing the wave function and energy levels with a longer tail in the initial wave function and a moving well.]
Time evolution

Wave function

Potential

Energy (MeV)

\(|\Psi|^2\)

x (fm)
Time evolution

Wave function

Potential

already some transfer at large separation
Time evolution

Wave function

Potential

|\psi|^2

Energy (MeV)

x (fm)

break-up component

T = 100/210
Time evolution

Wave function

Potential

large break-up component

Energy (MeV)

x (fm)
Time evolution

Wave function

Potential

break-up component
Time evolution (final)

Wave function

Potential

One node

N=2

N=1

N=2

break-up component

Energy (MeV)

x (fm)

T = 210/210

0

0

-20

-40

0

-5

-10

-15

-20
Time evolution (final)

Wave function

Continuum Q-value distribution
Q-value dependence (weakly-bound case)

The width of the Q-value window depends on the velocity of the moving well (acceleration).

**OBS:** Width of Q-value window depends on the velocity of the moving well (acceleration).
Another case: competing inelastic, transfer and break-up

Potential
Time evolution

Current

Wave function

Potential

\[ |\psi|^2 \]

\[ \text{Energy (MeV)} \]

\[ x \text{ (fm)} \]
Time evolution

Current

Wave function

Potential

Energy (MeV)

x (fm)
Time evolution

Current

Wave function

Potential
**Time evolution**

Current

Wave function

Potential

![Graphs showing current, wave function, and potential over time.](image-url)
<table>
<thead>
<tr>
<th>Final population</th>
<th>Exact</th>
<th>CC (with target continuum)</th>
<th>CC (with projectile continuum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>elastic</td>
<td>29 %</td>
<td>10 %</td>
<td>84 %</td>
</tr>
<tr>
<td>inelastic</td>
<td>7 %</td>
<td>16 %</td>
<td>3 %</td>
</tr>
<tr>
<td>Transfer gs</td>
<td>1 %</td>
<td>0.6 %</td>
<td>0.04 %</td>
</tr>
<tr>
<td>Transfer excited</td>
<td>16 %</td>
<td>36 %</td>
<td>0.2 %</td>
</tr>
<tr>
<td>Break-up</td>
<td>47 %</td>
<td>44 %</td>
<td>20 %</td>
</tr>
</tbody>
</table>

**OBS:** Inclusion of target continuum is fundamental
One more case
Time evolution

Current

Wave function

Potential

long tail in initial state

$|\psi|^2$

Energy (MeV)

$x$ (fm)
Time evolution

Current

Wave function

Potential

$|\psi|^2$

$E$ (MeV)

$x$ (fm)
Time evolution

Current

Wave function

Potential

$T = 1500$ of 3390

long tail effect
Time evolution

Current

Wave function

Potential

Energy (MeV)

$x$ (fm)
Time evolution

Current

Wave function

Potential

28% transfer?
<table>
<thead>
<tr>
<th>Final population</th>
<th>Exact</th>
<th>CC (with only bound states)</th>
<th>CC (with bound and continuum target states)</th>
<th>CC (with bound and continuum states in both target and projectile)</th>
</tr>
</thead>
<tbody>
<tr>
<td>elastic</td>
<td>20.7 %</td>
<td>95 %</td>
<td>21.4 %</td>
<td>21.0 %</td>
</tr>
<tr>
<td>transfer</td>
<td>0.3 %</td>
<td>5 %</td>
<td></td>
<td>0.04 %</td>
</tr>
<tr>
<td>transfer to Virtual state?</td>
<td>28 %</td>
<td></td>
<td></td>
<td>21 %</td>
</tr>
<tr>
<td>Break-up</td>
<td>51 %</td>
<td></td>
<td>78.6 %</td>
<td>57 %</td>
</tr>
</tbody>
</table>
Systems with two active particles (e.g. with two particles outside a closed shell or with a two-particle halo behavior)

We move now to the case of two active particles. From the pure mean-field approach we enter into the domain of the residual pairing interaction. Again we will consider a simple one-dimensional Case, with two particles initially in the target well and subject to the action of a second moving well. The relatively simplicity of the models allows to investigate in an easy way the effect of the pairing correlations on the space correlation, the influence of the continuum in the weakly bound “borromean” halo systems, the role of pairing in two-particle transfer processes and the still debate reaction mechanism for this process (sequential transfer ruled by the average field or “cluster-like” transfer?)
Two-neutron transfer

to test pairing interaction

Hamiltonian:

\[ \mathcal{H}(x_1, x_2, t) = KIN + V_T(x_1, x_2) + V_P(x_1, x_2, t) + V_{int}(x_1, x_2) \]

Density dependent residual interaction

\[ V_{int}(x_1, x_2) = -V \left[ \frac{\rho[(x_1 + x_2)/2]}{\rho_0} \right] \delta(x_1 - x_2) \]

(acting only when the two particles are both inside the same well)
Two-particle correlated wave function: correlation clearly favors the situation with the two particles on the same side

\[ |\Phi (x_1, x_2)|^2 \]
For comparison the situation with uncorrelated wave function

$|\Phi(x_1, x_2)|^2$
We can now follow the evolution in time of the two-particle wave function in the $(x_1,x_2)$ plane, due to the action of the moving well.

At the end of the process, from the final two-particle wave function, we can separate different final states:

1. elastic/inelastic (both particles still in the initial well)
2. one-particle transfer (one particle in the initial well and one in the moving one)
3. one-particle break-up (in particle in the continuum outside the wells and one in the initial or final well)
4. two-particle transfer (both particles in the moving well)
5. two-particle break-up (both particles outside the wells)
Moving well
initial wave function

\[ \psi(x) = \psi_0 + a t^2 \]

| \psi|^2

Energy (MeV)

\[ x_p = x_0 + a t^2 \]
We first consider the case of uncorrelated systems (no residual interaction).

In this case the transfer process is induced by the mean-field of the moving well, and in terms of reaction mechanism the two-particle transfer can only be interpreted as obtained by the successive transfer of single particles.

In such a situation, in a perturbative approach, we expect a pair transfer probability

\[ P_2 \sim (P_1)^2 \]

Let us see what comes from the “exact” solution
Initial wave function

Final wave function

Perturbative estimate:

\[ P_2 = 0.2 \times 0.2 = 0.04 \]

OK
$$|\Psi(x_1, x_2)|^2$$

Position of the moving well

1 particle transfer

1 particle break-up

2 particle transfer

WEAK

STRONG

UNCORRELATED CASE
Let us see now the case of the “correlated” pair, due to the action of the residual pairing interaction (density dependent, therefore only acting when both particles are within a well, and not when the particles are “in the air”).
Perturbative uncorrelated estimate:
$P_2\text{ (uncorr)} = 0.26 \times 0.26 = 0.07$
$P_2\text{ (corr)} = 0.13 = 2 \times P_2\text{ (uncorr)}$

Pairing enhancement factor: 2
CORRELATED CASE

\[ \left| \Psi(x_1, x_2) \right|^2 \]

position of the moving well

1 particle break-up

2 particle break-up

1 particle transfer

2 particle transfer

STRONG

WEAK

1 particle break-up

1 particle break-up

1 particle break-up

2 particle break-up
The perturbing interaction (that produces the transfer process) is a one-body field (i.e. acting individually on each of the two particles). In the standard perturbative scheme this implies at least a second-order process to produce the pair transfer. The enhanced two-particle transfer originates from the correlation in the two-particle wave function, and not from the reaction mechanism.

PS A similar effect (enhancement of the two-step process with respect to the one-step due to the pairing correlation) is also present in two-particle break-up processes, two-particle knock-out and two-particle decay.
Thanks to Laura Moschini, Curro Perez Bernal, Kouichi Hagino, Antonio Moro and Jose’ Antonio Lay
Piran and Padova: birth (1692) and death (1770) of the great composer Giuseppe Tartini (his most famous work is The Devil’s Trill Sonata for violin)