Transport phenomena in superfluid neutron stars

Laura Tolós
ICE (CSIC-IEEC)
FIAS, University of Frankfurt

Superfluidity and pairing phenomena from cold gases to neutron stars
ECT*, Trento, Italy, 20 to 25 March 2017
Transport coefficients describe the response of the system to some external perturbation.

(Shear and Bulk) viscosity: resistance to gradual deformation by shear stress or tensile stress.

Thermal conductivity: property of a material to conduct heat.

Electrical conductivity: measures a material's ability to conduct an electric current.
**Core of neutron stars**

**Outer core** extends in a density range $0.5 \rho_0 \leq \rho \leq 2\rho_0$ and can be several kilometers deep. It is mainly composed of neutrons with some admixture of protons, electrons and muons. Neutron superfluidity and proton superconductivity are expected.

**Inner core** can be several kilometers in radius and have a central density as high as $10^{-15} \rho_0$. The composition and equation of state of the inner core are poorly known.

Possible scenarios:

- **Matter made of n,p,e,µ**
- **Matter made of n,p,e,µ,Y (=hyperons)**
- **Matter made of quarks**
- **+ Superfluidity/Superconductivity**

Fridolin Weber
Matter made of $n, p, e, \mu$
electrons and muons (lightest and most mobile particles) and neutrons (most abundant)

**Shear viscosity** $\eta = \eta_{e\mu} + \eta_n$.

- $e\mu$ collisions with protons: Flowers and Itoh '76 '79; Cutler & Lindblom '87
  Shternin and Yakovlev '08

- $n$-$n$ and $n$-$p$ collisions: Flowers and Itoh '79, Cutler and Lindblom '87,
  Benhar and Valli '07, Shternin and Yakovlev '08.

**Bulk viscosity**
direct and modified Urca processes in the core of a vibrating neutron star by Sawyer'89; Haensel & Schaeffer'92;
Haensel, Levenfish & Yakovlev '00 '01

**Superfluidity**:
- proton superconductivity affects $e\mu p$ interaction
- neutron superfluidity, phonon-phonon contribution (Manuel and LT '11,'13) and/or
  electron-phonon contribution (Bedaque & Reddy '13, Reddy et al'14)

**Conclusions**:
- $\eta_{e\mu}$ generally dominates over $\eta_n$
- $\eta$ is comparable with $\xi$ at $T \approx 10^8$ K and dominates at low $T$
- proton superconductivity increases the importance of $\eta$ in comparison with $\xi$
- neutron superfluidity: phonon-phonon/electron contributions might become important
**Matter made of n,p,e,\(e,\mu\)**

**Thermal conductivity** \(\kappa \approx \kappa_b + \kappa_{e\mu}\)

\(\kappa_b\): Baiko, Hensel & Yakovlev ‘01, …
\(\kappa_{e\mu}\): Flowers & Itoh ‘76 ‘79; Gnedin & Yakovlev ‘95; Shternin & Yakovlev ’07

**Conclusions:**
- \(\kappa_{e\mu} \geq \kappa_n\) for \(T \geq 2 \times 10^9\) K in normal matter and for higher \(T\) in superconducting matter
- in neutron superfluid matter:
  phonon contribution might be important (Manuel & LT ‘14) and/or electron-phonon (Bedaque & Reddy ‘13, Reddy et al’14)

![Graph showing temperature dependence of thermal conductivity](image)
Possible observations to constrain transport properties of the core

- shear and bulk viscosities inside the neutron star core can help to understand r-mode instability and emission of gravitational waves

Data for accreting pulsars in binary systems (LMXBs) vs instability curves for nuclear and hybrid stars.

Alford & Schwenzel '13

- heat conduction problem in neutron star cores is needed to model cooling of neutron stars (especially in the first 100 years of their life), thermal relaxation of pulsars after glitches,… (taken from Shternin & Yakovlev '07 and references herein)
Transport phenomena in superfluid neutron stars

Laura Tolós
Cristina Manuel, Sreemoyee Sarkar and Jaume Tarrús

- EFT and superfluid phonons
- EoS for superfluid neutron star matter
- Shear viscosity and the r-mode instability window
- Bulk viscosity
- Thermal conductivity
- Summary

Manuel and Tolos, Physical Review D 84 (2011) 123007
Manuel and Tolos, Physical Review D 88 (2013) 043001
Manuel, Tarrus and Tolos, JCAP 1307 (2013) 003
Manuel, Sarkar and Tolos, Physical Review C 90 (2014) 055803
EFT and superfluid phonon

Exploit the **universal character of EFT at leading order** by obtaining the effective Lagrangian associated to a superfluid phonon and implement the **particular features of the system**, associated to the coefficients of the Lagrangian, via the **EoS**

\[
\mathcal{L}_{\text{LO}} = P(X)
\]

\[
X = \mu - \partial_t \varphi - \frac{(\nabla \varphi)^2}{2m}
\]

non-relativistic case

Applicable in superfluid systems such as cold Fermi gas at unitary, \(^4\text{He}\) or neutron stars

Son ‘02
Son and Wingate ‘06
EoS for superfluid neutron star matter

In order to obtain the speed of sound at T=0 and the different phonon self-couplings one has to determine the EoS for neutron matter in neutron stars.

A common benchmark for nucleonic EoS is APR98
Akmal, Pandharipande and Ravenhall '98

which was later parameterized in a causal form
Heiselberg and Hjorth-Jensen '00

\[
\frac{E}{A} = \epsilon_0 y \frac{y - 2 - \delta}{1 + \delta y} + S_0 y^\beta (1 - 2x_p)^2
\]

\[
y = \frac{n}{n_0} \quad x_p = \frac{n}{n_0}
\]

For $\beta$-stable matter made up of neutrons, protons and electrons, the speed of sound at T=0 is

\[
\sqrt{\frac{\partial P}{\partial \rho}} \equiv c_s
\]

\[
\begin{align*}
n_0 &= 0.16 \text{ fm}^{-3} \\
\epsilon_0 &= 15.8 \text{ MeV} \\
\delta &= 0.2 \\
S_0 &= 32 \text{ MeV} \\
\beta &= 0.6
\end{align*}
\]
Effective Lagrangian for superfluid phonon at LO

\[ \mathcal{L}_{LO} = \frac{1}{2}((\partial_t \phi)^2 - v_{ph}^2(\nabla \phi)^2) - g((\partial_t \phi)^3 - 3 \eta_g \partial_t \phi(\nabla \phi)^2) + \lambda((\partial_t \phi)^4 - \eta_{\lambda,1}(\partial_t \phi)^2(\nabla \phi)^2 + \eta_{\lambda,2}(\nabla \phi)^4) + \cdots. \]

with \( \Phi \) the rescaled phonon field, and where the different phonon self-couplings can be expressed in terms of the speed of sound at \( T=0 \)

\[ v_{ph} = \sqrt{\frac{\partial P}{\partial \mu}} = \sqrt{\frac{\partial P}{\partial \rho}} \equiv c_s \]

and derivatives with respect to mass density:

Escobedo and Manuel ’10

\[ g = \frac{1 - 2u}{6c_s\sqrt{\rho}}, \quad \eta_g = \frac{c_s^2}{1 - 2u}, \quad \lambda = \frac{1 - 2u(4 - 5u) - 2w\rho}{24c_s^2\rho}, \quad \eta_{\lambda,1} = \frac{6c_s^2(1 - 2u)}{1 - 2u(4 - 5u) - 2w\rho}, \quad \eta_{\lambda,2} = \frac{3c_s^4}{1 - 2u(4 - 5u) - 2w\rho} \]

Results valid for neutrons pairing in \( ^1S_0 \) channel and also valid for \( ^3P_2 \) neutron pairing if corrections \( \tilde{\Delta}(^3P_2)^2/\mu_n^2 \) are ignored. Bedaque, Rupak and Savage ‘03
Including NLO corrections in the phonon dispersion law

\[ E_p = c_s p (1 + \gamma p^2) \]

\[ \gamma = - \frac{v_F^2}{45\Delta^2} \]

\( v_F \): Fermi velocity
\( \Delta \): gap function

\( \gamma < 0 \): first allowed phonon scattering are binary collisions

taken from parameterizations of different models (with \( {}^3P_2 \) gap at least 0.1 MeV)
Andersson, Comer and Glampedakis ‘05
Shear viscosity due to superfluid phonons

The shear viscosity is calculated using variational methods for solving the transport equation as

\[ \eta = \left( \frac{2\pi}{15} \right)^4 \frac{T^8}{c_s^8} \frac{1}{M} \]

where M represents a multidimensional integral that contains the thermally weighted scattering matrix for phonons.
Shear viscosity due to binary collisions of phonons scales as $\eta \propto 1/T^5$ (also for $^4$He and cold Fermi gas at unitary) while the coefficient depends on EoS.

**Mean free path** of phonons: establish when phonons become hydrodynamic

$$ l = \frac{\eta}{n \langle p \rangle} $$

$\langle p \rangle$: thermal average

$n$: phonon density

Alford, Braby, and Mahmoodifar ‘10
r-mode instability window (only shear)

\[-\frac{1}{\tau_{GR}(\Omega)} + \frac{1}{\tau_{\eta}(T)} = 0\]

Dissipation due to superfluid phonons start to be relevant at $T \approx 7 \times 10^8$ K for $1.4 \, M_\odot$ and $T \approx 10^9$ K for $1.93 \, M_\odot$

\[
\frac{1}{|\tau_{GR}(\Omega)|} = \frac{32 \pi G \Omega^{2l+2}}{c^{2l+3}} \frac{(l - 1)^{2l}}{((2l + 1)!!)^2} \left( \frac{l + 2}{l + 1} \right)^{2l+2} \int_0^R \rho r^{2l+2} dr
\]

\[
\frac{1}{\tau_{\eta}(T)} = (l - 1)(2l + 1) \int_{R_c}^R \eta r^{2l} dr \left( \int_0^R \rho r^{2l+2} dr \right)^{-1}
\]

$l=2$ (dominant)
Bulk viscosities due to superfluid phonons

The bulk viscosity coefficients are calculated from the dynamical evolution of the phonon number density or, equivalently, by using the Boltzmann equation for phonons in the relaxation time approximation.

\[ \zeta_i(\omega) = \frac{1}{1 + \left( \omega I_1^2 \frac{\partial \rho}{\partial \mu} \frac{T}{\Gamma_{ph}} \right)^2} \frac{T}{\Gamma_{ph}} C_i , \quad i = 1, 2, 3, 4 \]

\[ C_1 = C_4 = -I_1 I_2 , \quad C_2 = I_2^2 , \quad C_3 = I_1^2 , \]

\[ I_1 = \frac{60T^5}{7\zeta(3) - 7\zeta(5)} \left( c_s \frac{\partial B}{\partial \rho} - B \frac{\partial c_s}{\partial \rho} \right) , \quad B = c_s \gamma \]

\[ I_2 = -\frac{20T^5}{7\zeta(3) - 7\zeta(5)} \left( 2Bc_s + 3\rho \left( c_s \frac{\partial B}{\partial \rho} - B \frac{\partial c_s}{\partial \rho} \right) \right) , \]

Three independent coefficients:

\[ \zeta_1 = \zeta_4 \quad \zeta_1^2 \leq \zeta_2 \zeta_3 \quad \zeta_2, \zeta_3 \geq 0 \]

In the static limit

\[ \zeta_i = \frac{T}{\Gamma_{ph}} C_i , \quad i = 1, 2, 3, 4 , \]

* Khalatnikov
Phonon decay rate for phonon number changing processes: $2 \leftrightarrow 3$

$$\Gamma_{ph} = \int d\Phi_5(p_a, p_b, p_d, p_c, p_f) \| A \|^2 f(E_a) f(E_b) (1 + f(E_d)) (1 + f(E_c)) (1 + f(E_f))$$

with one 4-phonon vertex and one 3-phonon vertex

with only 3-phonon vertices
\( \xi_2 \) at \( n \geq 4n_0 \) is within 10% of the static value for \( T \leq 10^9 \) K and for the case of maximum values of the \( ^3P_2 \) gap \( > 1 \) MeV, while, otherwise, the static solution is not valid. **Bulk viscosity coefficients strongly depend on the gap.**

Compared to the contribution of Urca (also modified Urca) processes to the bulk viscosities in neutron stars, those are dominated by phonon-phonon processes.
The thermal conductivity relates the heat flux with the temperature gradient

\[ q = -\kappa \nabla T \]

and is calculated using variational methods* for solving the transport equation as

\[ \kappa \leq \left( \frac{4a_1^2}{3T^2} \right) A_1^2 M_{11}^{-1} \]

\[ a_1 = \frac{4c_s^4}{15\Delta^2}, \quad A_1 = \frac{256\pi^6}{245c_s^9} T^9 \]

where \( M_{11} \) is the (1,1) element of a N x N matrix (N determined by convergence). Each element is a multidimensional integral that contains the thermally weighted scattering matrix for phonons:

* Braby, Chao and Schafer '10
Need of NLO corrections in phonon dispersion law (seen for He$^4$ and CFL)
Perform a variational calculation up to $N=6$ (deviation from $N=5 \leq 10\%$). We find that as in CFL

$$\kappa \propto \frac{1}{\Delta^6}, \quad T \lesssim 10^9 \text{ K}$$

Mean free path of phonons (different from shear mean free path)

$$l = \frac{\kappa}{\frac{1}{3}C_U C_S} \quad l \propto 1/T^3$$

Thermal conductivity in neutron stars is dominated by phonon-phonon collisions

$10^{25} \lesssim \kappa_{\text{ph}} \lesssim 10^{32} \text{ erg cm}^{-1} \text{ s}^{-1} \text{ K}^{-1}$ from $0.5n_0$ to $2n_0$ up to $T_c$
Summary

Starting from a general formulation for the collisions of superfluid phonons using EFT techniques, we compute the shear and bulk viscosities as well as the thermal conductivity in terms of the EoS of the system (and the gap function).

- Binary collisions of phonons produce a shear viscosity that scales with $1/T^5$ (universal feature seen for $^4$He and cold Fermi gas at unitary) while the coefficient depends on EoS (microscopic theory).
  - $r$-mode window modified for $T \geq (10^8-10^9)$ K due to phonon shear viscosity.

- Bulk viscosity coefficients strongly depend on the gap and they are dominated by phonon-phonon collisions as compared to Urca (modified Urca) processes.

- Thermal conductivity due to phonons scales as $1/\Delta^6$, the constant of proportionality depending on the EoS. As compared to electron-muon collisions, phonon-phonon collisions dominate the thermal conductivity.

Need of an accurate analysis of electron-phonon collisions Bertoni, Reddy and Rrapaj '15.