Effects of QCD critical point on electromagnetic probes

Akihiko Monnai (IPhT, CNRS/CEA Saclay)
with Swagato Mukherjee (BNL), Yi Yin (MIT) +
Björn Schenke (BNL) + Jean-Yves Ollitrault (IPhT)

Phase diagram of strongly interacting matter:
From Lattice QCD to Heavy-Ion Collision Experiments
December 1, 2017, ECT*, Italy
Probing the QCD equation of state at finite density
(including effects of QCD critical point on electromagnetic probes)

Akihiko Monnai (IPhT, CNRS/CEA Saclay)
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Phase diagram of strongly interacting matter:
From Lattice QCD to Heavy-Ion Collision Experiments
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Introduction

- The QCD phase diagram is not well known at finite $\mu_B$
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- Determine the properties of QCD matter at finite $T$, $\mu_B$
- Verify the existence of a QCD critical point (QCP)
Introduction

- Beam energy scan: an experimental exploration of QCD phases

- Performed and planned at
  - RHIC (BNL)
    - Phase I (2009-11): 7.7-62.4 GeV
    - Phase II (2017-20?): 3.0 GeV?
  - HADES, FAIR (GSI)
  - NICA (JINR)
  - SPS (CERN)
  - J-PARC-HI (J-PARC)
  - etc.
## Method

- Possible approaches

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Akihiko Monnai (IPhT), Probing the QCD equation of state at finite density
Viscous hydrodynamic model

- for connecting the EoS and observables

Hydrodynamic equations are closed with the EoS that characterize thermodynamics of the system.
At vanishing density ($\mu_B=0$)

Equation of state (EoS)

- At vanishing densities

We have good lattice calculations of the QCD EoS

Is it what we see in heavy-ions collisions?
Equation of state (EoS)

- We systematically generate variations of EoS:

![Graphs showing variations of EoS with different colors representing different EOSs: EOS A, EOS B, EOS L, EOS C for the first graph and EOS D, EOS E, EOS L, EOS F for the second graph. The graphs display the relationship between pressure over temperature squared (P/T^4) and temperature (T) in GeV.]

- Low temperature side (T < 140 GeV) is constrained by hadron resonance gas for the Cooper-Frye freeze-out.
Mean $m_T$ and $dN/dy$

- Observables sensitive to the EoS
  
  - Entropy density $\leftrightarrow (1/R_0^3)dN/dy$
    
    $s(T_{\text{eff}}) = a \frac{1}{R_0^3} \frac{dN}{dy}$
    
    “Number/entropy per volume”
  
  - Energy density over entropy density $\leftrightarrow$ mean $m_T$
    
    $\frac{\epsilon(T_{\text{eff}})}{s(T_{\text{eff}})} = b\langle m_T \rangle$
    
    “Energy per number/entropy”

$T_{\text{eff}}$: effective temperature of the medium

$R_0$: effective radius of the medium where $R_0^2 \equiv 2 \left( \langle |x(\tau_0)|^2 \rangle - |\langle x(\tau_0) \rangle|^2 \right)$

$a$, $b$: constant factor
Hydrodynamic results

- (2+1)D ideal hydro, Au-Au, 0-5%, Glauber initial conditions

There is a good correspondence between the EoS and observables; a single set of (a,b) fits hydro results onto the EoS
Hadronic decay and viscosity

- $<m_T>$ vs. $(1/R_0^3)dN/dy$ plot for EoS L

- $(a,b)$ can be determined so that all the EoS are satisfied
Hadronic decay and viscosity

- $<m_T>$ vs. $(1/R_0^3)dN/dy$ plot for EoS L

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- Hadronic decay reduces $<m_T>$ and increases $dN/dy$
Hadronic decay and viscosity

- $<m_T>$ vs. $(1/R_0^3)dN/dy$ plot for EoS L

- $(a,b)$ can be determined so that all the EoS are satisfied
- Hadronic decay reduces $<m_T>$ and increases $dN/dy$
- Shear and bulk viscous effects cancel on $<m_T>$, increase $dN/dy$
Comparisons to experimental data

- Viscous hydro results w/ decays

- Compatible with the lattice QCD equation of state within errors
Comparisons to experimental data

- Viscous hydro results w/ decays

- Compatible with the lattice QCD equation of state within errors
- Larger effective # of degrees of freedom allowed by the data
At finite density ($\mu_B \neq 0$)

AM, S. Mukherjee and Y. Yin, in preparation
Equation of state ($\mu_B \neq 0$)

- Lattice QCD EoS: Taylor expansion

\[
\frac{P_{\text{lat}}}{T^4} = \frac{P_0}{T^4} + \frac{1}{2} \chi^{(2)}_B \left( \frac{\mu_B}{T} \right)^2 + \frac{1}{4!} \chi^{(4)}_B \left( \frac{\mu_B}{T} \right)^4 + \mathcal{O} \left( \frac{\mu_B}{T} \right)^6
\]
Equation of state ($\mu_B \neq 0$)

- Lattice QCD EoS: Taylor expansion

$$\frac{P_{\text{lat}}}{T^4} = \frac{P_0}{T^4} + \frac{1}{2} \chi_B^{(2)} \left( \frac{\mu_B}{T} \right)^2 + \frac{1}{4!} \chi_B^{(4)} \left( \frac{\mu_B}{T} \right)^4 + \mathcal{O} \left( \frac{\mu_B}{T} \right)^6$$

- Match to hadron resonance gas EoS at low T
Equation of state ($\mu_B \neq 0$)

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- Match to hadron resonance gas EoS at low $T$

  ▶ Taylor expansion is not reliable when $\mu_B/T$ is large
Equation of state ($\mu_B \neq 0$)

- Lattice QCD EoS: Taylor expansion

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- Match to hadron resonance gas EoS at low $T$
  
  - Taylor expansion is not reliable when $\mu_B/T$ is large
  
  - The **EoS of kinetic theory** must match the **EoS of hydrodynamics** at freeze-out for energy-momentum/net baryon conservation

\[ E_i \frac{dN_i}{d^3 p} = \frac{g_i}{(2\pi)^3} \int_{\Sigma} p_i^\mu d\sigma_{\mu,i} f_i \]
Equation of state ($\mu_B \neq 0$)

- Lattice QCD EoS: Taylor expansion

$$\frac{P_{\text{lat}}}{T^4} = \frac{P_0}{T^4} + \frac{1}{2} \chi_B^{(2)} \left( \frac{\mu_B}{T} \right)^2 + \frac{1}{4!} \chi_B^{(4)} \left( \frac{\mu_B}{T} \right)^4 + \mathcal{O} \left( \frac{\mu_B}{T} \right)^6$$

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  - The EoS of kinetic theory must match the EoS of hydrodynamics at freeze-out for energy-momentum/net baryon conservation
Equation of state ($\mu_B \neq 0$)

- We connect the equations of state as

\[
\frac{P}{T^4} = \frac{1}{2} \left[ 1 - \tanh \frac{T - T_c(\mu_B)}{\Delta T_c} \right] \frac{P_{\text{HRS}}(T, \mu_B)}{T^4} + \frac{1}{2} \left[ 1 + \tanh \frac{T - T_c(\mu_B)}{\Delta T_c} \right] \frac{P_{\text{lat}}(T_s, \mu_B)}{T_s^4}
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$$

- Conditions of monotonous increase of thermodynamic variables

$$
\frac{\partial^2 P}{\partial T^2} = \frac{\partial s}{\partial T} > 0, \quad \frac{\partial^2 P}{\partial T \partial \mu_B} = \frac{\partial s}{\partial \mu_B} = \frac{\partial n_B}{\partial T} > 0, \quad \frac{\partial^2 P}{\partial \mu_B^2} = \frac{\partial n_B}{\partial \mu_B} > 0
$$
Equation of state ($\mu_B \neq 0$)

- We connect the equations of state as

$$\frac{P}{T^4} = \frac{1}{2} \left[ 1 - \tanh \frac{T - T_c(\mu_B)}{\Delta T_c} \right] \frac{P_{\text{HRS}}(T, \mu_B)}{T^4} + \frac{1}{2} \left[ 1 + \tanh \frac{T - T_c(\mu_B)}{\Delta T_c} \right] \frac{P_{\text{lat}}(T_s, \mu_B)}{T_s^4}$$

- It is not a priori clear if this is satisfied because

$$\frac{\partial^2 P}{\partial T^2} = \frac{\partial s}{\partial T} > 0, \quad \frac{\partial^2 P}{\partial T \partial \mu_B} = \frac{\partial s}{\partial \mu_B} = \frac{\partial n_B}{\partial T} > 0, \quad \frac{\partial^2 P}{\partial \mu_B^2} = \frac{\partial n_B}{\partial \mu_B} > 0$$

- Conditions of monotonous increase of thermodynamic variables

$$\frac{\partial^2}{\partial T^2} \left[ 1 \pm \tanh \left( \frac{T - T_c}{\Delta T_c} \right) \right] = -2\Delta T_c^{-2} \tanh \left( \frac{T - T_c}{\Delta T_c} \right) \cosh^{-2} \left( \frac{T - T_c}{\Delta T_c} \right)$$

is negative above $T_c$
Equation of state \((\mu_B \neq 0)\)

- The result

\[
\frac{P}{T^4} = \frac{1}{2} \left[ 1 - \tanh \frac{T - T_c(\mu_B)}{\Delta T_c} \right] \frac{P_{\text{HRS}}(T)}{T^4} \\
+ \frac{1}{2} \left[ 1 + \tanh \frac{T - T_c(\mu_B)}{\Delta T_c} \right] \frac{P_{\text{lat}}(T_s)}{T_s^4}
\]

where

\[
T_c = 0.166 - 0.4 \times (0.139\mu_B^2 + 0.053\mu_B^4) \\
T_s = T + 0.4 \times [T_c(0) - T_c(\mu_B)]
\]

⚠️ Hydro model needs \(T_c > 0.16\) GeV for switching to UrQMD; Lattice agreement is at \(T_c < 0.14\) GeV
Equation of state \((\mu_B \neq 0)\)

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We can also introduce a test “critical point”
Equation of state ($\mu_B \neq 0$)

The result

$$\frac{P}{T^4} = \frac{1}{2} \left[ 1 - \tanh \frac{T - T_c(\mu_B)}{\Delta T_c} \right] \frac{P_{\text{HRS}}(T)}{T^4} + \frac{1}{2} \left[ 1 + \tanh \frac{T - T_c(\mu_B)}{\Delta T_c} \right] \frac{P_{\text{lat}}(T)}{T^4}$$

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We can also introduce a test “critical point”
QCD critical point (QCP)

- A landmark in the QCD-land (which may or may not exist)
QCD critical point (QCP)

- What observables are sensitive to a QCP?
  - Proposed so far:
    - Event-by-event fluctuation of multiplicities
    - Rapidity slope of directed flow $v_1$
    - etc.

We consider the effects of viscosity evolution on the QCP by looking at dileptons
Electromagnetic probes

For probing the critical point

- The QCP can leave imprint via critical behavior of viscosity in two ways:
  1. Modification of bulk evolution
  2. $\delta f$ correction in the emission rate

The QGP is EM transparent

Photons and dileptons know the history of the time evolution

Graphics by AM
Near the critical point

- Bulk viscosity becomes dominant

- Shear viscosity: \( \eta = \xi^{(4-d)/19} \)

- Bulk viscosity: \( \zeta = \xi^3 \)

- Baryon diffusion: \( D_B = \xi^{-1} \)

We focus on bulk viscosity in this study
Near the critical point

- Bulk viscosity becomes dominant
  
  - Shear viscosity: \( \eta = \frac{\xi^{(4-d)}}{19} \)
  
  - Bulk viscosity: \( \zeta = \xi^3 \)
  
  - Baryon diffusion: \( D_B = \frac{1}{\xi} \)
  
  We focus on bulk viscosity in this study

- Parameterization for bulk viscosity
  
  \[ \zeta = \zeta_0 \left( \frac{\xi_{eq}}{\xi_0} \right)^3 \]

  where \( \zeta_0 = 2 \left( \frac{1}{3} - c_s^2 \right) \frac{e + P}{4\pi T} \)

  A. Buchel, PLB 663, 276 (2008)

  \( \xi \) is parameterized based on Ising model
Near the critical point

- Bulk viscosity becomes dominant

- Relaxation time

\[ \tau_{\Pi} = \tau_{\Pi,0} \left( \frac{\xi_{eq}}{\xi_0} \right)^3 \]

is motivated by

\[ \lim_{k \to \infty} \frac{d\omega}{dk} = \sqrt{c_s^2 + \frac{\zeta}{\tau_{\Pi}(\epsilon + P)}} < 1 \]

- Causal hydro is applicable when \( \Pi \) is “frozen” at large \( \tau_{\Pi} \)

- The relaxation time from an AdS/CFT approach

\[ \tau_{\Pi,0} = C_{\Pi} \frac{18 - (9 \ln 3 - \sqrt{3}\pi)}{24\pi T} \quad (C_{\Pi} = 1) \]

is free of cavitation \((P + \Pi > 0)\)
Dileptons

- How is the emission rate affected

- $\delta f$ correction in the emission rate

\[
\frac{dN}{d^4x} = \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} f_1(E_1) f_2(E_2) \sigma(M) v_{rel}
\]

\[
= \frac{dN_0}{d^4x} + \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} [f_1^0(E_1) \delta f_2(E_2) + (1 \leftrightarrow 2)] \sigma(M) v_{rel}
\]

QGP $q^+q^- \rightarrow l^+l^-$

Hadron $\pi^+\pi^- \rightarrow l^+l^-$
Dileptons

- How is the emission rate affected

- $\delta f$ correction in the emission rate

$$\frac{dN}{d^4x} = \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} f_1(E_1) f_2(E_2) \sigma(M) v_{\text{rel}}$$

$$= \frac{dN_0}{d^4x} + \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \left[ f_1^0(E_1) \delta f_2(E_2) + (1 \leftrightarrow 2) \right] \sigma(M) v_{\text{rel}}$$

(1) Equilibrium emission rate

K. Kajantie, J. Kapusta, L. McLerran, A. Mekjian, PRD 34, 2746
Dileptons

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\]

(1) Equilibrium emission rate

\[
\frac{dN_0}{d^4xdM^2d^2p_Tdy} = \frac{\sigma(M)}{2(2\pi)^5} \frac{M^2}{2} e^{-E/T} \left( 1 - \frac{4m_a^2}{M^2} \right)
\]

(2) Off-eq. emission rate needs $\delta f_i$

K. Kajantie, J. Kapusta, L. McLerran, A. Mekjian, PRD 34, 2746
Dileptons

- Bulk viscous corrections

- Grad moment method (Israel-Stewart) in Boltzmann approx.

\[
\delta f^i = - f_0^i \left[ b_i D_\Pi E_i + B_\Pi (m_a^2 - E_i^2) + \tilde{B}_\Pi E_i^2 \right] \Pi
\]

The off-equilibrium rate is

\[
\frac{d\delta N}{d^4xdM^2d^2p_Tdy} = -\frac{\sigma(M)}{2(2\pi)^5} M e^{-E/T} \left( 1 - \frac{4m_a^2}{M^2} \right) \\
\times 2\Pi \left[ B_\Pi m_a^2 \frac{M}{2} + (\tilde{B}_\Pi - B_\Pi) \frac{M^3}{8} \right]
\]

\(B_\Pi\) and \(\tilde{B}_\Pi\) can be calculated in kinetic theory as functions of \(T\) and \(\mu_B\)
Dileptons

- Invariant mass spectra: w/ 1+1 D non-boost invariant hydro

```
M (GeV) 0.4 0.6 0.8 1 1.2 1.4

10^{-1} dN/dMdy (GeV^{-1})

Preliminary

Y = 0

ideal
bulk (w/ δf)
QCP (w/ δf)

Y = 2

ideal
bulk (w/ δf)
QCP (w/ δf)
```

- Bulk viscosity enhances low M spectra; the QCP can be visible

QGP: parton gas w/ \( m_{th} \)
Hadron: pion gas

*Results are sensitive to the form of δf, \( m_{th} \), # of hadronic components, etc.*
Summary

- Equation of state can be probed in heavy-ion collisions
  - $\mu_B = 0$
    - There is a mapping between $\varepsilon/s$ vs. $s$ and $<m_T>$ vs. $dN/R^3dy$
    - The data is compatible with lattice QCD EoS, more # of DOF may also work
  - $\mu_B \neq 0$
    - EoS is constructed using lattice QCD and resonance resonance gas
    - If there is a critical point, its signal may appear in invariant mass spectra of dileptons

- Future prospects
  - Systematic probing of the EoS at finite density
    - Changing the location of test QCP, comparing to the BES data, etc.
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Fine

- Grazie per l'attenzione!
Where do the QCD matter go through?

- in heavy-ion collisions

- Much more broadly spread in event-by-event hydrodynamics (up to $\mu_B$ around 0.7 GeV)

Akihiko Monnai (IPhT), Probing the QCD equation of state at finite density