Toward an effective field theory approach in energy density functional theory

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ECT* workshop
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Equation of state of neutron matter at N^2LO.

Strong dependence on V!
(cannot do sym. matter yet.)

S. Gandolfi, talk in ESNT workshop, 2017
Take another expansion
EDF

- Energy density functional (EDF) framework gives reasonable results at mean field, when sufficient amount of parameters (~10) are included.

**But,…**

- Include *more parameters won’t necessarily help*.

→ Limited predictive power.

**Is there a way to do EFT ?** (need to go beyond mean field to perform the test).
Turn off nucleon-nucleon d.o.f.,
Also, no EFT/ERE to guild the power counting

In term of power counting: Just like turn of the light in a cave.
Turn off nucleon-nucleon d.o.f.,
no EFT/ERE to guild the power counting

In term of power counting: Just like turn of the light in a cave.
First hint: a special case where an EFT expansion is known to work\ns
Pure neutron matter at very low density ($k_N a < 1$, $\rho < 10^{-6}$ fm$^{-3}$).

Lee & Yang formula (1957) describes the dilute system.

$\Rightarrow$ Can be re-derived by EFT with matching to ERE


$$\frac{E_{NM}}{A} = \frac{\hbar^2 k_N^2}{2m} \left[ \frac{3}{5} K.E. + \frac{2}{3\pi} (k_N a) + \frac{4}{35} (11 - 2 \ln 2) (k_N a)^2 \right] + O(k_N^3)$$

Expansion in $k_N a$

But the valid $\rho$ is way too low!
If take physical value of $a = -18.9$ fm, then impossible to fit pure neutron matter EoS outside region $k_F a \ll 1$ (adding $t_1, t_2, t_3$ terms doesn’t help).

Diagrams gives $V$ up to $O(k_F^8)$

- $-iC_0$
- $-iC_2 \frac{k^2+k'^2}{2}$
- $-iC_2' k \cdot k'$

<table>
<thead>
<tr>
<th>$\rho$ [fm$^{-3}$]</th>
<th>$E/A$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>200000</td>
</tr>
<tr>
<td>0.10</td>
<td>400000</td>
</tr>
<tr>
<td>0.15</td>
<td>600000</td>
</tr>
<tr>
<td>0.20</td>
<td>800000</td>
</tr>
</tbody>
</table>

$10^5$ $10^6$

Treatment: Re-sum

To be valid at higher $\rho$, $(k_N a)$ needs to be re-summed. (Steele (2000), Schafer (2005), Kaiser (2011))

$$\frac{E_{NM}}{N} = \frac{\hbar^2 k_N^2}{2m} \left[ \frac{3}{5} + \frac{2}{3\pi} \frac{k_N a}{1 - 6k_N a(11 - 2 \ln 2)/(35\pi)} \right]$$

Neutron matter only
Very dilute regime
YGLO: Resumed-inspired functional


\[ V = \frac{B_\beta}{1 - R_\beta \rho^{1/3}} + \frac{C_\beta \rho^{2/3}}{\text{higher order in L&Y to be resumed}^*} + \frac{D_\beta \rho^{2/3}}{\text{velocity-dep term}^*} + \frac{F_\beta \rho^\alpha}{3^+ - \text{body}} \]

\( B_\beta, R_\beta \) are fixed to reproduce first two term in Lee & Yang.

\[ \Rightarrow B_\beta = 2\pi \frac{\hbar^2}{m} \frac{v-1}{\nu} a_\beta, \quad R_\beta = \frac{6}{35\pi} \left( \frac{6\pi^2}{\nu} \right)^{1/3} (11 - 2\ln 2) a_\beta. \]

(degeneracy: \( \nu = 2(4) \) for \( \beta = \frac{0}{\nu}, \frac{1}{\nu} \))

\( a_0 = -18.9 \text{fm,} \quad a_1 = -20 \text{ fm} \).

\( \frac{E}{A} = KE_\beta + \frac{B_\beta \rho}{1 - R_\beta \rho^{1/3} + C_\beta \rho^{2/3}} + D_\beta \rho^{5/3} + F_\beta \rho^{\alpha+1} \)
Able to describe both sym and pure neutron matter EoS up to $2\rho_0$ very well with only 4 free parameters each.

Up to $\rho=0.3$ fm$^{-3}$
Asymmetric case

Parabolic approximation

\[
\frac{E_\delta}{A}(\rho) = \frac{E_{\text{sym}}}{A}(\rho) + S(\rho)\delta^2,
\]

\[
(\delta = (\rho_N - \rho_p)/(\rho_N + \rho_p))
\]

\[
L = 3\rho_0 \left( \frac{dS}{d\rho} \right)_{\rho=\rho_0}
\]

Before: Lots of models fail


FIG. 4: Symmetry energy at saturation density as a function of its slope \(L\). The black lines delimit the phenomenological area constrained by the experimental determination of the electric dipole polarizability in \(^{208}\text{Pb}\). The blue dotted lines delimit the area constrained by the same measurement in \(^{68}\text{Ni}\), and the red dashed lines refer to the measurement done in \(^{120}\text{Sn}\). The yellow area is the overlap. Inset: density dependence of the Symmetry energy for the two YGLO parametrizations of this work.
Asymmetric case

Our result (prediction)

Satisfies the experimental constraint.

FIG. 4: Symmetry energy at saturation density as a function of its slope $L$. The black lines delimit the phenomenological area constrained by the experimental determination of the electric dipole polarizability in $^{208}$Pb. The blue dotted lines delimit the area constrained by the same measurement in $^{68}$Ni, and the red dashed lines refer to the measurement done in $^{120}$Sn. The yellow area is the overlap. Inset: density dependence of the Symmetry energy for the two YGLO parametrizations of this work.
Second hint: Unitarity limit

D Lacroix, A. Boulet, M. Grasso, C. J. Yang, submitted to prc

• Scale invariance tells \( \frac{E}{E_{FG}} = \xi \) (Bertch parameter)

Nuclear system \((a=-18.9 \text{ fm})\) is close to unitarity.

• \(|a| >> R\) (range of interaction) \( \frac{1}{|a_s|} \frac{1}{k_F} \Rightarrow 4 \times 10^{-6} < \rho < 0.002[\text{fm}^{-3}]\)

• Functional contains resum of \((a_s k_F)^{-1}\):

\[
\frac{E}{E_{FG}} = 1 - \frac{U_0}{1 - (a_s k_F)^{-1} U_1} \\
+ \frac{R_0(r_e k_F)}{[1 - R_1(a k_F)^{-1}][1 - R_1(a_s k_F)^{-1} + R_2(r_e k_F)]}
\]

No free parameters: \(U_i, R_i\) come from QMC data (with \(V_{\text{unitarity}}\))
Lesson:

- Nuclear (many-body) systems are not too far from the unitarity limit.
- Just a few more parameters might be sufficient to describe data up to $\rho=0.3\ \text{fm}^{-3}$, this explains why Skyrme works!
How to establish an EFT with a Skyrme-like interaction?
What will an EFT-based force look like?

• Leading order (LO): Need to make a guess.
• Based on renormalizability analysis
  C.J. Yang, M. Grasso, U. van Kolck, and K. Moghrabi, coming soon!

⇒ A good guess would be the $t_0$-$t_3$ model (or $t_0$ model, but it gives a very bad EOS).

### Estimation of Breakdown scale

If require $O\left(\frac{k_F}{M_{hi}}\right)^1 > O\left(\frac{k_F}{M_{hi}}\right)^2$ to be valid up to $\rho = 0.3$ fm$^{-3}$. Then $M_{hi}$ need to be at least 400 MeV.

Also, the low bound cannot do better than the unitarity limit.

Then, only applicable for $\rho > 4 \times 10^{-6}$[fm$^{-3}$].
Diagrammatic explanation of How Skyrme works
Dressing of propagator → $V_{\text{eff}}$

Leading order (LO)

Then, NLO includes:

\[ V_{\text{NLO}}^{\text{eff}} \]

\[ V_{\text{LO}}^{\text{eff}} G V_{\text{LO}}^{\text{eff}} \]

\[ \text{diverge at least in } \Lambda k_F^3 \]

\[ V_{\text{Sly}^5}^{\text{eff}} G V_{\text{Sly}^5}^{\text{eff}} \]

Dressing of propagator $\rightarrow V_{\text{eff}}$

Leading order (LO)

Then, NLO includes:

* $V_{\text{eff}}^{\text{NLO}}$ contains (at least) contact terms to renormalize $V_{\text{eff}}^{\text{LO}} G V_{\text{eff}}^{\text{LO}}$. 
Counter term part of the NLO potential

\[ V_{\text{eff}}^{NLO} : \] For \( t_0 - t_3 \) model, the divergence from \( V_{\text{eff}}^{LO} G V_{\text{eff}}^{LO} \) is:

\[ O(k_F^3), O(k_F^{3+3\alpha}), O(k_F^{3+6\alpha}). \]

If want to keep \( \alpha \) free, \( \Rightarrow \) Minimum contact term required: \( Ck_F^{3+6\alpha} \).

Most general case: \( A k_F^3, B k_F^{3+3\alpha}, C k_F^{3+6\alpha} \).

In infinite matter, \( k_F^{3n} \) in-distinguishable with \( 3\pi^2 \rho \)

\( \Rightarrow k_F^n \) -term in EOS could originated (at interaction level) from \( (k - k')^{3n} \rho^\nu \), where \( \nu \) is an extra parameter to be decided in the fitting to finite nuclei.
NLO results (based on $t_0$-$t_3$ as LO)

$\alpha<1/6$ case*

C.J. Yang and M. Grasso, coming soon!

* For $\alpha>1/6$, $V_{\text{eff}}^{\text{NLO}}$ also includes $t_1$,$t_2$ terms.
Renormalization group (RG) check

Prescription: $B^{(*)}, C^{(*)}$

Prescription: $C^{(*)}$
How to apply to finite nuclei

- One simple version of beyond mean field interaction has been applied via PVC (with the phonon replaced by p-h pair).

- In principle, a general refitting is needed. One either perform the fit directly in the chosen beyond mean field scheme, or use subtraction.

- To be fully consistent, n parameters in the interaction means n subtractions are needed.
Future prospects

Try to bridge EFT ideas/techniques to mean field (and beyond) within EDF framework.

Mean field with potential models (effective interaction).
(e.g., Skyrme-type)

2nd order corrections

Add new effective interactions?
What is the proper form of it?
Is the improvement systematic?

Higher order corrections

Goal: Systematic treatment of the interactions.

Renormalization-group analysis + power counting check
Thank you
2nd order correction (symmetric & neutron matter)

\[
\frac{E}{A} = \frac{E^{(0)}}{A} + \frac{E^{(2)}}{A} + \ldots
\]

\[
\frac{E_{\text{sym}}^{(2)}}{A} = \frac{-3m^*}{64\pi k_F^3 (2\pi)^6} \sum_{S,T} (2T+1)(2S+1) \int_{C_1} d^3k_1 \int d^3k_2 \int d^3q [vGv]
\]

\[
G = \frac{1}{q^2 + q \cdot (k_1 - k_2)}
\]

Contour of integral (\(C_1\)):

| \(k_{1,2} \in [0, k_{F_{1,2}}] \)

| \(k_1 + q \gg k_{F_1}, k_2 - q \gg k_{F_2} \)
Results for nuclear matter


\[
\frac{\Delta E^{(2)}_{\text{sym}(l=0)}}{A} = - \frac{mk^6_p}{110880 \hbar^2 \pi^4} \left\{ \begin{array}{c}
-6534 + 1188 \ln [2] + 3564 \lambda - 19602 \lambda^3 - 5940 \lambda^5 \\
+ (1782 - 20790 \lambda^4) \ln \left[ \frac{\lambda^2-1}{\lambda^2+1} \right] \\
+ (24948 \lambda^5 - 5940 \lambda^7) \ln \left[ \frac{\lambda^2-1}{\lambda^2+1} \right] \\
- 14996 + 2112 \ln [2] + 5280 \lambda - 2860 \lambda^3 \\
- 48840 \lambda^5 - 18480 \lambda^7 + (2640 - 55440 \lambda^6) \ln \left[ \frac{\lambda^2-1}{\lambda^2+1} \right] \\
+ (71280 \lambda^7 - 18480 \lambda^9) \ln \left[ \frac{\lambda^2-1}{\lambda^2+1} \right] \\
- 9886 + 1128 \ln [2] + 2520 \lambda + 147 \lambda^2 - 3654 \lambda^5 \\
- 35280 \lambda^7 - 15120 \lambda^9 + (1260 - 41580 \lambda^6) \ln \left[ \frac{\lambda^2-1}{\lambda^2+1} \right] \\
+ (55440 \lambda^9 - 15120 \lambda^{11}) \ln \left[ \frac{\lambda^2-1}{\lambda^2+1} \right] \\
\end{array} \right\} \tilde{T}_{03}^{2} \tilde{T}_{03}^{1} \tilde{T}_{1}^{2} \tilde{T}_{1}^{1} \\
\]

Diverge as $\Lambda^5$

\[
\frac{\Delta E^{(2)}_{\text{sym}(l=1)}}{A} = - \frac{mk^6_p}{73920 \hbar^2 \pi^4} \left\{ \begin{array}{c}
-1033 + 156 \ln [2] + 420 \lambda + 140 \lambda^3 - 840 \lambda^5 \\
- 5880 \lambda^7 - 2520 \lambda^9 + (-210 + 6930 \lambda^8) \ln \left[ \frac{\lambda^2-1}{\lambda^2+1} \right] \\
+ (9240 \lambda^9 - 2520 \lambda^{11}) \ln \left[ \frac{\lambda^2-1}{\lambda^2+1} \right] \\
\end{array} \right\} \tilde{T}_{2}^{2} \tilde{T}_{2}^{1} \tilde{T}_{1}^{2} \tilde{T}_{1}^{1} \\
\]

Diverge as $\Lambda^5$

\[
\frac{\Delta E^{(2)}_{\text{neutr}(l=0)}}{A} = - \frac{mk^4_{FN}}{166320 \hbar^2 \pi^4} \left\{ \begin{array}{c}
-6534 + 1188 \ln [2] + 3564 \lambda - 19602 \lambda^3 - 5940 \lambda^5 \\
+ (1782 - 20790 \lambda^4) \ln \left[ \frac{\lambda^2-1}{\lambda^2+1} \right] \\
+ (24948 \lambda^5 - 5940 \lambda^7) \ln \left[ \frac{\lambda^2-1}{\lambda^2+1} \right] \\
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Diverge as $\Lambda^5$

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\end{array} \right\} \tilde{T}_{2}^{2} \tilde{T}_{2}^{1} \tilde{T}_{2}^{1} \tilde{T}_{1}^{2} \tilde{T}_{1}^{2} \tilde{T}_{1}^{1} \\
\]

Diverge as $\Lambda^5$
PART II:
RENORMALIZABILITY
• When $\Lambda \rightarrow \infty$, how the 2nd order terms behaves?

$$\frac{\Delta E_f^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4\hbar^2} k_F^4 \left[ A_0 + A_1 T_3 k_F^{2\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4 \right],$$  \hspace{1cm} \text{Converge terms}

$$\frac{\Delta E_a^{(2)}(k_F, \lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[ B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2 \right],$$  \hspace{1cm} \text{Diverge, } k_F\text{-dep appears in MF}

$$\frac{\Delta E_d^{(2)}(k_F, \lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[ C_0 T_3^2 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4 \right],$$  \hspace{1cm} \text{Diverge, } k_F\text{-dep not in MF}
• Idea: Absorb the \( \Lambda \)-divergence in 2\textsuperscript{nd} order into mean field terms with the same \( k_F \)-dependence.

\[
\frac{\Delta E^{(2)}_f(k_F)}{A} = \frac{3m}{2\pi^4 \hbar^2} k_F^4 \left[ A_0 + A_1 T_3 k_F^{2\alpha} + A_2 T_3 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4 \right],
\]

converge

\[
\frac{\Delta E^{(2)}_\alpha(k_F, \lambda)}{A} = -\frac{m}{8\pi^4 \hbar^2 \lambda k_F^3} \left[ B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2 \right],
\]

Diverge, \( k_F \)-dep appears in MF

\[
\frac{\Delta E^{(2)}_e(k_F, \lambda)}{A} = \frac{m}{8\pi^4 \hbar^2 \lambda k_F^3} \left[ C_0 T_3 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4 \right],
\]

Diverge, \( k_F \)-dep not in MF

eliminate by setting \( \alpha=1/3 \) and \( t_1=t_2=0 \), or setting \( t_1=t_2=t_3=0 \).
• Idea: Absorb the $\Lambda$-divergence in 2nd order into mean field terms with the same $k_F$-dependence.

$$\frac{\Delta E_j^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4\hbar^2} k_F^4 \left[ A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4 \right], \quad \text{converge}$$

$$\frac{\Delta E_a^{(2)}(k_F, \lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[ B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2 \right], \quad \text{Diverge, } k_F\text{-dep appears in MF}$$

Treatment 1: Absorb divergence into redefinition of parameters.
Treatment 2: Add counter terms correspond to each divergence.